INFORMATION PROCESSING AND NUMBER

by

PAULINE M.de VERE

M.A. THESIS

Education Department
University of Canterbury

February 1990

ABSTRACT

A previous study suggested that children use their own informal problem solving methods based on counting procedures or mental arithmetic but lost or changed information in mental calculation. Children's counting skills and aspects of memory in relation to their ability to complete addition and subtraction problems were examined across a wide age range in the primary school.

The results showed that subjects with good mathematical ability had well developed schema about number and applied this in abstract processing of information when solving problems. Subjects with poor mathematical ability had little understanding of number, only procedural knowledge, and used concrete counting proedures in problem solving to find an answer.

Children's information processing skills are discussed in terms of memory , schema theory and the role of self concept in controlling mental processes.

A remedial teaching programme based on developing abstract information processing skills was trialled over a six week period with some success. The need for implementation of research findings into classroom programmes is advocated.

ACKNOWLEDGEMENTS

Thanks are due to Mr. David Hughes for his supervision of the project, and to Professor Graham Nuthall for his advice during discussion.

Miss Lorraine de Vere gave assistance with marking and photocopying material and her help is gratefully acknowledged.

Thanks are due to Mrs. Sue Rose for her efforts in collecting and distributing material among the school staff.

The children and teachers at the two schools involved in the pilot and main study deserve special thanks for their co-operation and support throughout the year.

Pauline M. de Vere

LIST OF TABLES

CHAPTER 1

INTRODUCTION

The previous research project conducted by the author with a class of Standard two pupils found that school taught written methods were not used in solving arithmetic problems. The children preferred to count actual items or to use mental arithmetic based on counting methods. Two interesting observations were made. Occasionally totals counted were forgotten necessitating a recount of items, sometimes two or three times before the total was recalled and recorded, and during mental calculation data was sometimes lost or changed. It seemed that as the number of steps involved in processing information increased the load on working memory became too great and hence data was lost. These observations in turn raise questions about information processing and a possible limited working memory capacity.

The aims of this study were to investigate the relationship between information processing and the ability to solve addition and subtraction problems and then to design, implement and evaluate a remedial teaching programme based on deficiencies identified in children's processing skills.

1.1 Development of Addition and Subtraction Strategies

Personal observation of stategies employed by a number of children from all class levels in primary schools has suggested some children across a wide age range continue to use counting on fingers as an aid to computation. According to Ginsburg (1977) school arithmetic is assimilated into the informal methods, based on counting procedures, which children previously developed as a means of solving everyday practical problems. Informal methods were found to persist in spite of instruction in schools (Ginsburg,1982). This may lead to a bizarre arithmetic where procedures are mechanically followed without understanding of the principles involved.

Hughes (1983) has attributed this difficulty with school arithmetic to children's inability to translate between concrete and formal representations. This is an important factor in learning the written code and in solving arithmetic problems long after the code has been acquired. One study conducted by Stallard and described by Hughes (1986) concluded that understanding of the formal code was related to children's mathematical ability rather than their age or amount of instruction received at school.

Carpenter and Moser (1982) suggested that children's difficulty in analysing and solving problems can be traced in part to the transition from using informal problem solving strategies to memorised number facts and algorithms. The findings from all these studies suggest that children's difficulties with the formal code may be the result of mechanically operating on the numbers given without regard for the problem structure or understanding of the operation involved.

The research conducted suggests that informal counting procedures form the basis of early addition and subtraction strategies. These develop out of school and persist in spite of school instruction (Ginsburg 1977). Similar findings have been reported across cultures (Saxe,1985; Ginsburg,H. 1982) and social class suggesting that the development of counting skills is a natural developmental process like language acquisition.

1.2 Counting and Number Operations

A number of studies have described the number competence of pre school and school entrants in such areas as counting and solving simple word problems (Siegler and Robinson,1982: Groen and Resnick,1977; Hughes,1983; Young-Loveridge,1987). The spontaneous invention of the counting on method following instruction on the counting all method was described in a study by Groen and Resnick (1977) and Hughes (1981) noted children's success on simple verbal problems but suggests that their difficulty with school arithmetic is due to their inability to translate between concrete and formal codes. It seems that children develop a practical arithmetic based on counting procedures which they use in everyday situations but they may have some difficulty in adapting to written school methods. The implementation of the Beginning School Mathematics Programme (1985) into New Zealand schools involves a relatively late introduction to the formal code of arithmetic and an emphasis on pre number skills such as classification and logic while counting is regarded as an unimportant rote activity.

Counting has been described as a rule governed system which enables the generation of any number in that system (Pollio and Whitacre 1970). There have been a number of studies of children's strategies used in addition and subtraction and these have included counting and noncounting methods. The type of units that children count has been described as a developmental sequence progressing from manipulation of concrete objects, to pointing, verbalising and finally operating at an abstract level using number facts (Steffe, von Glasersfeld, Richards and Cobb,1983).

The use of number facts was the final step in a counting model described by Carpenter and Moser (1982) which identified a progression in children's addition strategies from counting all from one, counting on from the first addend, and at a later stage the larger addend, to use of number facts. Children were also observed using known facts to solve unknown facts. Number facts used in this way were usually the "doubles" or "ties" and sums to ten.

Non-counting methods used in addition and described by Houlihan and Ginsburg (1981) included the use of memorised facts recalled directly from memory, memorised facts that are related to the numbers stated in the particular problem and used in indirect and novel ways, and place value with larger numbers. The use of known facts may thus be part of a progression that begins with counting concrete objects and develops to more abstract use of number as numeration skills begin to appear.

Studies that have investigated problem difficulty in addition with sums to ten have found that the "doubles" or "ties" were easier than expected (Groen and Parkman, 1972; Siegler and Robinson,1982). No explanation for this finding was given but from descriptions of children's addition strategies this type of fact is frequently used in solving other related facts. The ties or "doubles" form the two times multiplication table and one child was observed to use 2x4=8 in solving a subtraction problem 7-3=4 (de Vere ,1989). There have been few investigations of multiplication strategies. Newman, Friedman and Gockley (1987) examined children's early attempts to count in multiples and reported that five and six year old children were more likely to count in multiples on clustered displays of small numerosity and counting in twos was the most frequent type of counting in multiples.

In the study by de Vere (1989) children in their fourth year at school were found to use many of the strategies described above but the main finding of this study was a preference for mental arithmetic or counting objects over the formal code in problem solving. Subjects who used mental arithmetic relied on known facts and used the commutative property of addition and the inverse to obtain subtraction facts. However there was one major problem in this approach. Information was lost or changed as children worked their way through all the steps of a calculation. It appeared that working memory was overloaded.

1.3 Theories of Number Concept Development

Case (1978,1982) has proposed a theory of children's mathematical development concerned with changes in short term memory span. According to Case, children are initially only able to deal with one aspect of a problem at a time. As long as the task can be broken down into parts that can be attended to one at a time, for example counting and simple addition/subtraction , then children who are at the early stage of mathematical development will be successful. As processing becomes more efficient and more bits of information can be attended to at a time problems of increasing complexity may be solved successfully. The increase in working memory was not considered to arise from an increase in attentional capacity but rather from an increase in automaticity of the operations being performed. This theory still retained the stages of development that are a feature of Piaget's work on general cognitive development (1953).

Brainerd (1979) proposed a different approach, that the order of acquisition of number concepts was ordinal properties, operations, cardinal properties. Initially the place of numbers in the standard counting sequence is known, this knowledge is then applied in arithmetic which finally leads to an appreciation of the cardinality of numbers at a later age, about 10 years old. This model conflicts with Piaget's stages of development which stress the importance of number conservation and class inclusion before number work can become meaningful. Piaget's work has been subjected to recent criticism and review (Hughes 1986;

Young-Loveridge,1987) because it does not take account of those pre school number skills that have been identified in more recent research and it considers general cognitive development whereas number concept development involves many specific ideas.

More recently studies have suggested that early number knowledge is not as systematic as initially thought (Baroody,1987). Children seek efficiency in the procedures they use and so will look for short cuts. For example when adding 2+4, they will count on from the larger addend, 4, but this doesnot mean that the commutative property of addition is being applied and until the standard counting sequence is familiar children will always need to begin their count at one then later by moving up and down the number line they are able to perform operations without any understanding of the operations involved. Early knowledge is procedural rather than conceptual.

Among this recent research are information processing models of number concept development where new knowledge is built onto existing ideas (Schaeffer 1974) and Case's work on working memory capacity. This imposes a limit on information that can be processed not on account of limited storage space but due to an increased automacity of operations at one level opening up space to use more complex executive strategies. Tests of this capacity, M-space, must include storage and some transformation of data across an increasing number of items. Case (1978,1982) suggests that a certain minimum level of operational automaticity at one stage is a pre requisite for transition to the next step. This theory includes stages of development that are related to Piaget's stages. Criticism of Case's theory is centred on the ability to measure units of mental space and reliability is questioned as other experts would break down the tasks he uses into different steps (Flavell,1978).

Case's theory, however, could account for children with small M-space being unable to progess in number concepts until they attained the minimal level of operational automaticity and it could explain the loss of data observed during processing (de Vere,1989).

1.4 Memory and Mental Calculation

Studies involving learning disabled and slow learners show that they score poorly on memory tests and take a long time to make small gains in learning (Steffe,1983:Denvir and Brown, 1986, Baroody,1983,1987;Ceci 1984). A study by Baroody (1983) involving a boy with learning difficulties in mathematics showed that this subject had problems with basic facts. He preferred addition to subtraction, knew few subtraction facts and only knew a few of the doubles. Strategies used in computation included changing the order of addends.

On the other hand subjects with exceptional memory or mental calculation skills learn quickly and utilise their extensive knowledge base in combination with patterns and relationships between numbers, for example the squares of numbers are recalled and used in calculating other products (Hunter,1977;Chase and E ricsson,1981;Hope,1987).

Children appear to discover and utilise patterns in number operations such as the use of "doubles", changing the order of addends and the inverse relationship between addition and subtraction. This reduces the number of facts that have to be memorised but may also be important in uncovering the relationship between facts and the underlying basic principles involved in a rule governed number system.

This use of known facts and patterns suggests that these able students are able to call on their knowledge and experience stored in long term memory. Thus over time they have continued to develop a knowledge base through extensive practice and an interest in numbers.

Theories of Cognitive Development

Any model of information processing needs to consider both working and long term memory and their interaction. Chunking of data (Miller,1956) has been proposed as a mechanism where the capacity of working or short term memory could be increased. A capacity of seven chunks was suggested in this research but the size of each chunk could be a single item or a whole package of related information.In this way expert knowledge gained such as in chess (Chi,1978) can be utilised by seeing groups of pieces as a unit.

Chi (1978) discusses three factors that influence memory development; strategies, knowledge and capacity in terms of "chunks". By organising knowledge into chunks less capacity is used as each chunk constitutes a unit in working memory rather than individual pieces of information. Using memory for chess positions the skilled children retrieved more chunks than the novice adults. Brown and Deloache (1978) suggest the development of more efficient and effective strategies in organising thinking as a factor in memory development.

Memory models described in the literature are becoming more complex as they propose an articulatory loop, a visual and spatial component and a central control within working memory . These components are seen as the inner eye and inner ear.The inner ear or articulatory loop has been found to have a limited capacity of about 1.8 seconds (Baddley and Hitch 1976;Hulme and Tordoff,1989). Younger children speak more slowly than adults and older children and this may explain differences in recall across age groups when recall is limited to what each subject can say in about 2 seconds.

Hebb (1961) tested the notion that memory span may also involve long term memory using a digit span with repeated digits test, where the same nine digit number was repeated every third item in a test of 24 items. The results showed that subjects performed significantly higher on recall of this repeated item than on the other items in the test indicating that long term memory may be involved in digit span tests.

Prior knowledge is stored in long term memory. The way this knowledge is organised may be explained by schemas or frameworks of mental representation. New experiences are interpreted in terms of what is already known and experts such as the young chess players in Chi's study (1978) will have well developed schemas relating to knowledge in a specific area that provide a rich framework for assimilating incoming information.

A review of literature has shown children's information processing to be an area that has been investigated in understanding how number concepts develop. There is a need for more work in this field taking account of how all aspects of memory are involved.

A reliance on mental arithmetic in problem solving revealed some problems with working memory capacity and this requires a fuller investigation. There are children who have made little progress in spite of mathematical instruction throughout their years at primary school and who appear to be working at the early developmental level according to Case's theory. The traditional school methods have not met these children's needs and there is a need for instructional programmes based on an understanding of how number concepts develop in terms of schema theory and a limited working memory capacity.

CHAPTER 2

AN INFORMATION PROCESSING MODEL

2.1 Limited Capacity

Models of information processing are becoming more complex. One observation noted is the capacity of the auditory loop being what a subject can say aloud in just under two seconds. There are two points of particular interest here, the time appears to be a constant across a range of ages yet individual capacities vary as they depend on speech rate. Thus older subjects will have a greater capacity due to their faster speech rate. A number of processing models have included a category of memory that has limited capacity, in terms of limited storage capacity according to Atkinson and Shiffrin (1968), limited processing capacity according to Case (1982) and now a limited time span according to Baddley and Hitch (1974).

If incoming information can be retained for a constant time period this suggests some physical rather than mental function of information processing is involved. Incoming information is transmitted by neurons in the form of electrochemical impulses (Restak,1984). Sensory neurons carry information about the environment to the receptive areas in the brain. Like all types of waves electrical impulses will dampen or fade over time. This is actually an important characteristic as information channels have to be cleared so that further information can be processed or a block would form. The alternative is recall of all that has ever been received and this would pose enormous storage problems. The reception time of incoming data must be long enough to select data of interest for processing without blocking the whole system. A limited time capacity memory system would be consistent with this requirement.

2.2 Schema and Information Processing

Incoming information is processed and stored in long term memory. Schema theory proposes a structure for storing information so that it can be retrieved in flexible ways. Any model of memory storage ,encoding and retrieval must account for levels of processing in terms of declarative and procedural knowledge, episodic and semantic memory and the ability to form visual images (Cohen,Eyesenck,Le Voi,1986).

The organisation of all types of prior knowledge in a complex but flexible system is difficult to define but ideally should be as simple as possible. In nature the basic elements of the universe are time, space and matter but cognition also involves reasoning and procedural knowledge. All knowledge can be organised to include these features if a set of key questions are used as a way of accessing the store for entering or retrieving data. 'When' and 'where' cover time and space and these are important in episodic memory, 'what' and 'who' describe matter, 'why' covers reasoning and 'how' procedural knowledge. The 'why' question is important because it forms links between and within schema and leads to semantic memory for a given concept.

All six question groups will contain several items each of which will have their own schema so the whole intersecting structure very quickly becomes extrememly complex. In this way a schema is held together as a unit of knowledge but a specific line of inquiry may be pursued in depth by accessing a string of related schema. This is consistent with the idea of connecting links between schema resulting in spreading activation where the flow of ideas reflect a rich network of associations in long term memory.

For example the schema for 'toothbrush' may include plastic in the 'what category' as the material it is made of but a schema for plastic also exists. Why toothbrushes often have plastic handles connects the two schema: plastic is inexpensive, lightweight and easily cleaned and toothbrushes are frequently replaced, have to be held in the hand and rinsed after use. When and where a toothbrush was used could include episodic memory about cleaning teeth this morning and the method of using a toothbrush may be described as procedural knowledge.

2.3 Self Interrogation

If existing knowledge is organised in this manner then it may be accessed by asking questions. Brown and DeLoache(1978) suggest self interrogation skills are important in cognitive development. The use of prior knowledge in problem solving will require self questionning through internal rather than external speech. The activation of electrochemical impulses now originates in the brain rather than as a result of incoming sensory data that calls for a response. This involves thinking and experiments have been conducted where activity in the brain has been recorded for concious thinking (Restak, 1984). In an experiment where subjects were asked to flex their index finger at will a change in brain activity was found to occur 1.5 seconds before the subject flexed his finger. A stimulus response ordinarily takes about 0.1 seconds while here brain activation precedes the movement by 1.5 seconds demonstrating that internal processing has already taken place.

2.4 Self Schema

One schema of particular is interest is the 'self schema' (Markus,1980). The organisation of self knowledge will include personal attitudes, interests, and emotions all of which may influence the way a problem is tackled. Without interest or motivation it may not be attempted at all. It appears possible that the self schema could control all activity making decisions on which sensory data to attend to, production of visual images or accessing schema.

2.5 Proposed Model of Information Processing

A model of information processing is proposed that suggests short term or working memory is actually thinking either as internal communication involving silent speech or visual images. Its limited capacity is time based due to the nature of transmission of electrochemical impulses. When sensory information is involved as in immediate recall of data or in mental problem solving there an interaction between prior knowledge and incoming data. Thinking is a process that occupies time not space. More effective information processing will enable efficient use of the limited capacity available through chunking of information from schema in the long term memory store. This prior knowledge is always available through self interrogation in problem solving.

In terms of information processing in number operations subjects who are able to call on prior knowldege and apply the understanding they have built up within well developed schema should be able to use self questionning and internal or abstract reasoning. Until internal speech and self questionning is developed information processing will be procedural involving concrete items.

To test these hypotheses the questions to be answered in this study include:

1. What similarities/differences exist in subjects' information processing for different ability and age groups?
2. What types of information processing are used by the different subject groups?
3. How do differences in information processing relate to problem solving ability?
4. How do types of information processing explain ability differences in language/mathematics?
5. How can teachers help children develop abstract thinking skills?

CHAPTER 3

TEST DEVELOPMENT

3.1 Materials

A series of short tests were developed on the basis of the findings of a previous study (de Vere,1989) and related reseach in the areas of memory and number concept development. Tests were designed to investigate the relationship between counting skills, long term memory, short term memory, working memory and the ability to solve addition and subtraction problems.

I. Counting Sequence

According to Ginsburg (1977) children use informal methods based on counting procedures in problem solving and continue to do so in spite of school instruction. Counting is not a rote activity but involves the use of a rule governed system which enables a child to generate numbers at any point in the system. If the underlying structure is not fully understood then problems arise when subjects are asked to generate numbers at different points in the sequence. Several common stopping points (breaks in the standard counting sequence) have been observed and these reflect subjects' current knowledge.

Two tests were designed to investigate the ability to generate numbers in the standard counting sequence involving counting forwards and backwards.

Test A.Name the next counting number after a given number.
Test B.Name the previous counting number before a given number.

Individual items were based on common stopping points that reflect different levels of understanding of the structure of our standard counting sequence:

Level 1. One digit numerals
Level 2. Two digit numerals but not the end of a decade.
Level 3. Numerals at the end of a decade.
Level 4. Three digit numerals with no tens or ones digit.
Level 5. Three digit numerals, no tens and 9 as ones digit.

There is no pattern to the sequence of one digit numerals in Level 1 items and the standard sequence at this level involves rote learning. Failure at this level would indicate that this early learning has not yet been mastered. Level 2 items involve adjustment of one to the ones digit only and an understanding of the repetition of the 1- 9 cycle within each decade. The transition across a decade is a common stopping point with the incorrect decade following or an alternative offered. For example counting 28, 29, 20 10,20 11...This example demonstrates an appreciation of maintaining the tens digit and increasing the ones digit by 1 but fails to appreciate the place value structure of our number system where zero may be a place holder.

Three digit numerals introduce a further difficulty as there are now hundreds, tens and ones to consider. Level 4 items were included to identify those subjects who have difficulty with whole hundreds. A common error here is to increase the hundreds instead of the ones digit so subjects making this error would count 98,99,100,200,300... A particularly difficult transition is from numbers like 109 to 110 where there is a zero in the tens place. A common error here is to go to the next hundred, for example, 109,200,201,202,203...209,300,301....following the pattern established in the decades.

Subjects who fail at a particular level would be unable to complete the next level as their counting would break from the standard sequence before that point was reached.

II.Counting/Numeration

The common errors made in attempting to continue the standard counting sequence suggest difficulties with the place value structure of the number system. Children with good counting skills are able to use their knowledge of the underlying structure to generate any number in the system. There is also some evidence to suggest that an understanding of the place value structure is used in problem solving procedures developed by children to cope with everyday situations.

Informal problem solving methods used by children include a procedure where a tens first approach is used rather the ones first method taught in schools for calculations with two and three digit numbers. This involves operating on the tens and then the ones value. Resnick (1982 p146) describes an example of this type of processing where two digit numbers were partitioned into tens and ones and the two sets of values separately operated on beginning with the tens first.

Addition or subtraction of ten from numbers around common stopping points in the standard counting sequence should provide further evidence of difficulties with the place value structure underlying the number system.

Test C. Add ten to a given number.
Test D. Subtract ten from a given number.

Level 1. One digit numerals.
Level 2. Two digit numerals.
Level 3. Three digit numerals with no zeros.
Level 4. Three digit numerals with zero tens and ones.
Level 5. Three digit numerals around a hundred value.

Items at all levels can be completed by counting on ten from the number given but errors made will reflect those made in the counting sequence Tests A and B. Subjects who use the place value structure of a number and add on a ten to the incorrect digit will show a different type of error on Level 3, 4 and 5 items.

Subjects who count on ten from the number given will show errors that reflect their own counting system, for example 300 +10 would result in 1300 for subjects who increase the hundreds digit once three digit numerals are reached, but 400 if they count to 309 and go to the next hundred. If subjects are using the place value of each digit then addition of a ten by adjusting the hundreds instead of the tens digit will also result in 400 but addition of a ten to 294 will give 394, a Level 5 item. The answers for these two types of item will have to be jointly considered in order to determine the kind of processing that is involved.

III. Pattern Recognition and Continuation

Studies of subjects with expert memory and the ability to perform complex mental calculations have shown that these subjects frequently use patterns and relationships between numbers to aid their recall and computation. Their prior knowledge and expertise is available from long term memory and may be applied in many different situations.

Informal methods used by children rely on mental arithnetic and a test of pattern recognition and continuation was included to determine the role of long term memory in problem solving.

Test E. Name the next two numbers after a given sequence of four numbers.

There are many types of number pattern that could be included and careful consideration was given to the types of patterning that would reflect processing used in other tests:

Level 1. Repeating patterns.
Level 2. Common counting patterns.
Level 3. More difficult counting patterns.
Level 4. Common difference between members.
Level 5. Patterns within the digits.

In order to provide examples that required the types of processing used in other tests one of the two items at each level was an increasing sequence and the other item a decreasing sequence so that continuing the patterns included counting on or back and addition and subtraction of a common difference.

Items at Level 1 required recognition and recall of the pattern unit but no calculation whereas Level 2 and 3 items required recognition and then continuation of counting patterns (tens and twos from unfamiliar starting points, elevens and fives from familiar starting points). When counting objects children use counting in twos or tens as a more efficient way of dealing with large groups of objects and adding on or subtracting a ten is used in mental calculations so counting in tens or twos are well known counting patterns. To increase the difficulty of the items unfamiliar starting points were used for Level 2 items. Counting in elevens or fives, Level 3 items, is not as frequently used as counting in tens and twos but involves readily recognisable patterns when familiar starting points are used.

Counting in threes and fours are not easily continued from unfamiliar starting points and addition or subtraction of a common difference is more useful in continuing these patterns. Level 4 items with a common difference required subjects to identify and then apply the difference. Pattern recognition was more difficult in Level 5 items. These items could be completed by finding and applying a larger common difference or by recognising a pattern within the digits, for example, one digit increasing as the other digit decreased.

Over the five levels pattern recognition becomes more difficult and mental processing needed to continue the patterns increases from recall at Level 1, to counting at Levels 2 and 3, and addition or subtraction at Levels 4 and 5.

IV. Short Term Memory

One of the difficulties observed when subjects were recalling counted totals or performing mental arithmetic was remembering data (de Vere, 1989). Some subjects could not recall totals of objects they had just counted and had to count them again, one girl taking three counts before she could recall and record the total number of objects. Similarly subjects asked for data to be repeated as they had forgotten what was said when solving number problems. It appeared that some subjects had problems with recall from short term memory and if the data is not retained then further processing is clearly impossible. The subjects with recall difficulties were from the group of below average ability. The digit span test used in WISC-R test series was used as a measure of short term memory. Both digit span forwards and backwards tests were used and the five levels of items involved an increase in the number of digits to be recalled at each level..

Test F. Recall a series of digits in the correct order.
Test G. Recall a series of digits in reverse order.

Recalling digits in reverse order requires more than recall. Data has to be retained and then worked on before recalling the new order. Previous research has shown that not only was data lost but also changed whilst performing mental calculations. Test G would provide a measure of the ability to work on data and then recall new data.

V. M-Space (Working Memory Capacity)

The more steps involved in a calculation the more likely it is to cause problems when performing mental calculations. Case (1978, 1982) has proposed a theory of number concept development based on mental space capacity where children are initially only able to deal with one aspect of a problem then as working capacity increases more units of information can be attended to at a time. In order to measure mental space a test involving counting coloured spots and then recalling totals was used (Case, Kurland and Goldberg, 1982).

In this study a test that could be administered to whole classes was required. The procedure used by Case was adapted so that subjects counted and recalled totals of items at five levels, the number of totals to recall increasing at each level.

Test H. Recall a sequence of counted totals .

VI. Working Memory

Case did not include operations in his measure of mental space and this test was included to more closely match the processing involved in mental arithmetic where data has to be retained and operated on and then new data recalled. Subjects were given a sequence of digits and then asked to add one or two to each digit in the sequence and recall the new totals. The number of digits within items increased at each of the five levels included.

Test I. Add a constant to a series of digits and recall the new series of digits.

VII. Addition/Subtraction Problems

Initially it was hoped to include verbal problems as a measure of problem solving ability. However there are many factors that contribute to the difficulty of a verbal problem (syntactic, semantic and structural) and to take account of these factors would require a lengthy test. All problems can be reduced to a small number of written forms involving addition and subtraction. As the aim was to investigate subjects' ability to operate on numbers rather than interpret verbal problems items for these two tests involved completing number sentences for five levels of difficulty:

Test J. Record the missing numeral from each example in a series of addition number sentences.

Test K. Record the missing numeral from each example in a series of subtraction number sentences.

Level 1. Sums to ten.

Level 2. Sums between 10 and 18.

Level 3. Missing addend.

Level 4. Initial quantity unknown.

Level 5. Equivalence of two operations.

All items could be solved in more than one way: by counting methods, recall of basic facts or adjusting from known facts such as 6+6=12 so 6+7 is 1 more, that is 13. Items at different levels included items that could be solved by methods of increasing difficulty determined by the findings of research on strategies used in addition and subtraction (Carpenter and Moser, 1982). Even if basic addition facts have been acquired by rote learning Level 3, 4 and 5 items have to be changed to a form where recall of facts can be used so more than recall of facts is involved in solving these problems. Levels of difficulty for items are described in terms of counting methods using fingers as aids because no other help was available in the test situation.

Level 1 items could be solved by counting on fingers using a 'counting all' method but Level 2 items involved numbers above 10 and so fingers are not sufficient unless a 'counting on' method is used where fingers are used as a means of keeping track of how far the count has proceeded. Level 3 items with a missing addend also require 'counting on' methods with fingers keeping track of the count but now the number of fingers needed is the unknown. When the initial quantity is the unknown, as in Level 4 items, subjects have to change the order of addends in addition items, for example [ ]+2=3 becomes the same as 2+[ ]=3. The reversibility of operations has to be applied to change the subtraction number sentence to a form that can be solved by counting methods, for example [ ]-1=2 changes to 2+1=[ ]. Level 5 items may be solved by methods used for Level 4 items but only after the equivalence of the two operations is appreciated.

An understanding of the equals sign and the equivalence of the two operations in Level 5 items is essential if these items are to be completed. Research described by Hughes (1986) suggests that children interpret the equals sign as an instruction to put the answer next as happens when pressing that button on a calculator. Level 5 items were included to investigate subjects' understanding of the equivalence of the two operations.

No item involved numbers above 18 as interest was in processing information at a level all subjects could complete and Standard 1 pupils have little experience with calculations involving two and three digit numbers.

Completing items at different levels involved more steps in processing information as the levels increased. The different methods used at each level could be explained in terms of increasing mental working capacity as proposed by Case.

<u>VIII.Long Term Memory/Prior Knowledge</u>

Studies related to children's acquisition of basic facts have shown that some facts are learned before others. The doubles or ties such as 2+2 and addition of 1 are among the first facts learned. These known facts are often used to work out other facts (Carpenter and Moser,1983; Steinberg,1985) particularly the doubles where a problem like 3+4 is solved by saying 3+3 is 6 so 3+4 is 1 more, that is 7. Children have also been observed adding tens and hundreds, for example 10 +20 is 30 and 100 and 200 is 300, and use this in their own informal counting methods for solving two and three digit problems. Another commonly observed feature observed in addition strategies is changing the order of addends so that counting on from the larger addend occurs so that 2+9 becomes the same as 9+2. All these strategies help to make processing information more efficient and involve use of prior knowledge.

Wittrock's theory of generative learning (1974) where new knowledge is built on existing knowledge emphasises the importance of prior knowledge. In order to investigate subjects' prior knowledge in terms of the type of facts known and strategies developed a test was needed where subjects could relate the extent of their existing knowledge. Two alternatives were proposed for inclusion in the test series:

Test L.Write as many equations as possible in five minutes.

Write 'nine' in as many different ways as possible in five minutes.

Five levels of item difficulty were required if this test was to be consistent with other tests in the series. Research related to addition and subtraction strategies suggested possible levels of difficulty:

Level 1. Doubles and adding 1.
Level 2. Adding tens and hundreds, basic facts to 18
Level 3. Changing order of addends, harder add/sub.
Level 4. Application of rules and generalisations.
Level 5. Patterns and recall of related units of data.

Equations with doubles or adding 1 should appear at the easiest level as these are acquired first while demonstration of the effect of adding zero and recording number patterns within equations should be at higher levels as they require more abstract thinking.

These were only suggested categories for each level and a more detailed description of item level was to be completed after examining the types of equation recorded by subjects in the pilot study.

3.2 Test Presentation

How knowledge is stored in memory and later recalled is open to debate, some researchers favouring imagery and others schema where data is stored in related units. The way in which data is presented may have an effect on the way it is processed so two equivalent tests were produced one to be administered orally and the other visually.

A series of cards was made for the visual presentation test. These white cards, 10cm x10cm, had a black numeral or shape (for the counting totals Test H) in the centre. (See Appendix A for examples of the shapes used and examples of cards). Number sentences were displayed on similar longer pieces of white card. All the cards for a particular test were arranged in order and then held together with metal rings so that cards could be flipped over as they were exposed to the subjects. In order to ease card movement the bottom edges were cut so that the front card was always longer than the next card. This prevented two cards being flipped at once and made it easy to quickly flip the top card.

Instructions and items were written on the same sheet for the auditory test presentation with one exception. Test H, counting and recalling totals, was recorded on tape cassette as this would be too difficult to conduct in a test situation. The test items involved playing groups of sounds, with a one second interval between each sound , on a variety of instruments including drum, triangle, organ, whistle, recorder, wood block and bell. Sequence of sounds was arranged so that contrasting sounds could be clearly heard by the subjects. Test instructions for this test were also recorded on tape.

All tests were presented to subjects with a one second interval between each member of an item in tests where more than one number per item was involved (pattern recognition and memory tests). Between 10 and 20 seconds was allowed for recording answers. Details of test administration are given in Appendix B.

Typed answer sheets were prepared with five tests arranged down one half of the sheet and the remaining tests down the other half of the sheet. Sheets were folded down the centre as a means of ensuring subjects recorded their answers in the appropriate spaces. A sample item was included at the top of each test space and then the ten items were arranged in five pairs so that the two items at each level were on the same line for marking purposes. (An example of a completed answer sheet for the main study test is shown in Appendix C.) The back of the sheet was used for recording subjects' own equations in the prior knowledge test.

3.3 Item Difficulty

All sub-tests administered in the pilot study included ten items with two items at each one of five levels of increasing difficulty. On memory tests the difficulty increased as more digits were added to the sequence to be recalled. The difficulty of items on other tests was determined by the type of item at each level. The items included in each test are given in Table 1 for the auditory presentation. Auditory test items used in the main study are included in the details of test administration in Appendix B.

Table 1

Counting, Memory and Number Test Items-Auditory Test Presentation

---

Test A Counting Sequence

Name the number after:

S1. 3

Q1. 14 Q2. 19
Q3. 26 Q4. 74
Q5. 39 Q6. 89
Q7. 200 Q8. 500
Q9. 209 Q10. 509

Test B Counting Sequence

Name the number before:

S2. 8

Q11. 15 Q12. 19
Q13. 48 Q14. 74
Q15. 40 Q16. 89
Q17. 300 Q18. 500
Q19. 110 Q20. 510

Test C Counting/Numeration

Add 10 to these numbers:

S3. 5

Q21. 4 Q22. 9
Q23. 16 Q24. 52
Q25. 350 Q26. 190
Q27. 503 Q28. 109
Q29. 294 Q30. 691

Test D Counting/Numeration

Subtract 10 from these numbers:

S4. 13

Q31. 12 Q32. 17
Q33. 36 Q34. 87
Q35. 275 Q36. 461
Q37. 500 Q38. 700
Q39. 407 Q40. 605

Test E Pattern Recognition

Name the next two numbers:

S5. 10-20-30-40

Q41. 4-7-4-7 Q42. 9-1-9-1
Q43. 17-27-37-47 Q44. 59-57-55-53
Q45. 11-22-33-44 Q46. 50-45-40-35
Q47. 98-87-76-65 Q48. 19-28-37-46
Q49. 2-5-8-11 Q50. 98-94-90-88

Test F Digit Span

Recall in this order:

S6. 4-8

Q51. 3-8-6 Q52. 6-1-2
Q53. 3-4-1-7 Q54. 6-1-5-8
Q55. 8-4-2-3-9 Q56. 5-2-1-8-6
Q57. 3-8-9-1-7-4 Q58. 7-9-6-4-8-3
Q59. 5-1-7-4-2-3-8 Q60. 9-8-5-2-1-6-3

Test G Digit Span Backwards

Recall in reverse order:

S7 . 4-6

Q61. 2-5 Q62. 6-3
Q63. 5-7-4 Q64. 2-5-9
Q65. 7-2-9-6 Q66. 8-4-9-3
Q67. 4-1-3-5-7 Q68. 9-7-8-5-2
Q69. 1-6-5-2-9-8 Q70. 3-6-7-1-9-4

Test H Mental-Space

Recall totals in this order:

S8. 3-4

Q71. 5-2 Q72. 8-3
Q73. 4-7-2 Q74. 3-6-4
Q75. 6-3-5-2 Q76. 7-4-2-6
Q77. 3-5-4-7-2 Q78. 2-7-3-5-4
Q79. 7-4-3-5-2-6 Q80. 4-2-5-3-6-7

Test I Addition of a Constant:

Add the constant to these numbers:

S9. 3 +2

Q81. 5 +1 Q82. 7 +2
Q83. 7-4 +2 Q84. 5-3 +1
Q85. 4-6-3 +2 Q86. 3-1-4 +1
Q87. 2-5-6-4 +1 Q88. 6-3-7-4 +2
Q89. 4-1-8-3-5 +1 Q90. 5-2-3-1-7 +2

Test J Addition Examples:

Find the missing number:

S10. 6+2=[]

Q91. 3+4=[] Q92. 5+2=[]
Q93. 4+9=[] Q94. 5+6=[]
Q95. 9+[]=12 Q96. 5+[]=13
Q97. []+8=15 Q98. []+9=14
Q99. 2+9=4+[] Q100. 6+8=5+[]

Test K Subtraction Examples:

Find the missing number:

S11. 5-3=[]

Q!01. 9-5=[] Q102. 10-3=[]
Q103. 17-8=[] Q104. 11-7=[]
Q105. 15-[]=9 Q106. 12-[]=4
Q107. []-6=7 Q108. []-3=8
Q109. []-7=1+7 Q110. []-6=5+4

Test L. Prior Knowledge

Write as many equations as you can in five minutes.

3.4 Scoring Procedures

A method of scoring outlined by Case and used in his study of M-Space was chosen as the most appropriate method for this series of tests(Case,Kurland and Goldberg,1982). Interest was in mastery of the items at different levels rather than an overall total score on each sub- test.Each sub-test contained pairs of items at the same difficulty level and there were five levels of difficulty making a total of ten items. Scores from 0 to 5 indicate the subject's ability to succeed on the types of item at the five levels.

The recorded answers were marked correct or incorrect and the highest level where at least one of the two items was correct was located, and a total score given as shown in the following examples:

Level 1	Q.1	✓	Q.2	✓
Level 2	Q.3	✓	Q.4	x
Level 3	Q.5	✓	Q.6	✓
Level 4	Q.7	x	Q.8	x
Level 5	Q.9	x	Q.10	x

In the above example a score of 3 would be given as that is the last level where at least one of the two items was correct. As one item on Level 2 has been answered correctly the subject has been given credit for ability to master items at level 2.

Level 1	Q.1	✓	Q.2	✓
Level 2	Q.3	✓	Q.4	x
Level 3	Q.5	x	Q.6	✓
Level 4	Q.7	x	Q.8	x
Level 5	Q.9	x	Q.10	x

In this second example a score of 2.5 would be given as Level 3 is the highest level where at least one item has been answered correctly but no credit is given for the incorrect answer at this highest level. Any correct answers recorded after a level where both items were incorrect would receive a score of 0.5 for each correct response and this would be added onto the highest level score. Thus in the above example if Q.10 had also been correct a score of 3 (2.5 +0.5) would have been given.

3.5 Subjects

The subjects who participated in the pilot study, with an age range from 7.5 to 10 years, came from two parallel S1-S3 classes in an inner city Christchurch primary school. A total of 40 subjects ,18 girls and 22 boys (nine girls and eleven boys from each class) took two forms of the trial test.

Form A of the P.A.T. Mathematics was administered to the Standard 2 and 3 subjects early in March and analysis of the results showed a range from CPR1-CPR94. Eight children in each class were considered to be working at a below average level, C.P.R.<23 and one girl from one class and a boy and a girl from the other class were considered to be working at a high average level C.P.R.> 76 (descriptive terms for subject performance are based on those used in T.O.S.C.A.,1981 P.13). The small group of standard 1 children had shown a poor understanding of number concepts with the exception of one boy who worked with the Standard 2 group in class programmes. Each class thus covered a wide ability range but included a large group of below average ability children.

3.6 Procedure

The tests developed were trialled on pupils from two parallel S1-S3 classes (Group P and Group Q) in an inner city Christchurch primary school. The tests were administered in two forms to both classes. Items of similar difficulty were prepared for presentation in either auditory or visual form. One class had the auditory presentation test then the visual presentation test on the following day while the other class had the visual presentation followed by the auditory presentation. The researcher who was also a class teacher administered the visual presentation and the other class teacher administered the auditory presentation to each class. Time to complete test administration was over one hour and this was thought to be too long. In fact a break was taken before the additon/ subtraction examples which were completed on the same day after lunch.

3.7 Selection of Test Series for the Main Study

It was much easier to administer the auditory presentation because items and instructions were read from the same sheet whereas in the visual presentation cards had to be held up in addition to reading instructions and keeping to set times. The subjects occasionally missed a visual item when they did not look at the card as instructed but were able to hear all items even if they did not immediately focus on the administrator. During the visual presentation test subjects were observed reading from the cards and had to be asked not to read the numerals out aloud because they were distracting other subjects. By reading out the numerals subjects were putting the visual information into auditory form. The auditory test appeared to have some administrative advantages over the visual test presentation.

A summary of the mean scores and standard deviations for the pilot study tests is given in Table 2.

Table 2

Summary of the Pilot Study Test Results

Subjects		Test A	Test B	Test C	Test D	Test E	Test F	Test G	Test H	Test I	Test J	Test K
P and Q Auditory	Mean	4	3.5	2.5	1.5	1	2.5	1.5	0.5	1	2	1.5
	S.D	1.5	2	2	2	1.5	1.5	1	0.5	1.5	1.5	1.5
P and Q Visual	Mean	3.5	3	2.5	1.5	1	2	1.5	1	1	2	1.5
	S.D.	1.5	2	2	2	1.5	1.5	1	0.5	1.5	1.5	1.5
Correlation Auditory/Visual		0.79	0.76	0.86	0.91	0.69	0.55	0.4	0.11	0.56	0.76	0.76

Mean level scores for the two types of test presentation are within 0.5 level on all tests for both class groups, P and Q, and the whole sample of 40 subjects. Correlation coefficients between the two test presentations are all positive for each of the sub-tests but some sub-tests showed very little variance in final scores as all subjects' performance was biased towards one end of the scale. For example on the M-space test a low correlation of 0.11 was calculated because all subjects performed at low levels on both tests as shown in Table 3. Totals for the whole series of tests were calculated and correlation between the two test presentations was found to be 0.96. As the tests correlated highly it did not appear necessary to administer both forms of

Table 3

Distribution of Correct Responses for Auditory and Visual Tests

Item Level	Test A V. A.	Test B V. A.	Test C V. A.	Test D V. A.	Test E V. A.	Test F V. A.	Test G V. A.	Test H V. A.	Test I V. A.	Test J V. A.	Test K V. A.
0	3 4	7 8	5 10	18 13	23 20	5 5	7 8	11 15	16 15	8 12	13 13
0.5	1 1	0 0	4 2	1 5	1 9	4 4	3 0	11 15	5 3	0 3	2 3
1	2 0	4 0	8 4	4 8	5 4	5 0	8 5	9 8	8 8	7 2	4 8
1.5	1 2	1 2	1 1	1 0	0 2	4 2	9 4	6 0	5 2	0 1	1 1
2	0 0	2 0	1 3	3 2	1 0	9 7	1 8	0 1	1 3	5 4	6 3
2.5	0 0	0 0	3 0	0 0	1 0	2 1	5 6	3 0	2 5	2 4	1 1
3	8 4	4 4	1 5	1 2	2 2	6 9	6 7	0 1	0 1	8 5	7 6
3.5	2 2	0 1	1 3	3 0	1 0	2 4	0 1	0 0	1 0	3 2	1 0
4	0 2	4 2	6 4	2 3	3 0	1 6	0 0	0 0	0 1	5 4	4 5
4.5	1 0	0 0	3 2	4 2	3 0	0 1	1 0	0 0	2 2	0 0	1 0
5	22 25	18 23	7 6	3 5	0 3	2 1	0 1	0 0	0 0	2 3	0 0
Total	40 40	40 40	40 40	40 40	40 40	40 40	40 40	40 40	40 40	40 40	40 40

V.-Visual Presentation Test
A.-Auditory Presentation Test

the test and as the auditory presentation had advantages for both the subjects and the administrator this form of the test was selected for large scale testing in the main study at a different school .

Selection of tests to be included in the main study was based on the spread of scores on individual sub-tests. The distribution of correct answers for the two class groups on both forms of the test is given in Table 3.
Tests A and B appeared to be rather easy, with over half the subjects scoring at Levels 4 or 5, but they made a good introduction to the series of tests and identified those subjects with limited counting skills. Tests C, D, F, G, J and K gave a satisfactory range of scores and needed no change.

The pattern recognition test, Test E, appeared to give a low result. Although most subjects had performed poorly on this test inspection of the distribution of individual subject responses, given in Table 3, showed that a small group of able children had scored at higher Levels 4 and 5 and as the final test was to be administered to subjects from S1 to F2 it was decided to retain the test with some alteration to item order. The items in both Test E presentations were examined for difficulty and the final selection of ten items were arranged in order of difficulty. Level 4 items in the pilot test now became Level 5 items and Level 5 items became Level 4 items as more subjects had correctly answered Level 5 than Level 4 items in the pilot test. Auditory test items now appeared to include a suitable range of difficulty and they were retained in the new order in the main study tests.

Tests H and I produced low scores and took a relatively long time (ten minutes) to administer. A test of M-space was needed in the final test battery and Test H met this need so it was decided to replace Test I, addition of a constant, with the prior knowledge Test L in the final series of tests. This had the further advantage of reducing testing time. As the trial testing had only used Standard 1 to 3 subjects including few high average pupils Test H was not changed but it was decided that if a similar result occurred in the large scale testing further investigation of this test would be undertaken.

The prior knowledge Test L, where subjects wrote their own equations, provided a range of examples that could be grouped in five levels and this exercise was retained in the main test series, becoming the new Test I, but the alternative visual Test L, writing names for nine only provided a limited range of material and so was not included. Data from this test could not be scored in five levels so no results are given.

Number sentences written by the subjects were grouped into five different categories similar to those outlined in the test development section:

Level 1. Doubles (e.g. 2+2=4, or 7+7=14)
Addition of 1 (e.g. 2+1=3, 4+1=5)

Level 2. Counting and memorised facts
Simple addition/subtraction of basic facts
(e.g. 1+3=4, or 17-9=8)
Addition of tens/hundreds
(e.g. 30+50=80, or 100+200=300)
Writing out 2x, 5x, 10x tables

Level 3. Operations that use counting on procedures
Addition/Subtraction across ten/hundred
(e.g. 43+9=52, 105-8=93)

Pairs of equations

(e.g.3+4=7 and 4+3=7 or 8-2=6 and 8-6=2)

Addition and subtraction of a constant

(e.g. 9+2-2=9)

Level 4. Demonstration of rules/generalisations

Reverse operations (e.g.3+4=7 and 7-4=3)

Addtion of 1 and 0

(e.g. 135 234 +1=135 235 or 135 234+0=135 234)

Associative and distributive properties

(e.g. [5x2]+[4x6]=34)

Level 5. Recall of units /chunks of data

Patterns (e.g. 1+9=10,2+8=10....9+1=10)

Clusters around a number

(e.g. 0+1=1, 0x0=0,5-5=0,0+50=50)

The final series of tests consisted of the counting Tests A and B, the numeration/counting Tests C and D, pattern recognition Test E (with some modification), memory Tests F and G, M-space Test H and the addition/subtraction Tests J and K. On completion of these tests subjects were asked to show how much they knew about numbers by writing as many number sentences as possible in five minutes (Test I).

This series of tests was administered in auditory form to two parallel S4 to F2 classes from the same school as the pilot study subjects and testing time was found to be 45 minutes. No problems were encountered in test administration or marking and scoring so no further alteration was made before testing at a different school in the main study.

CHAPTER 4

METHOD

4.1 Subjects

The subjects in the main study came from ten classes in a full primary school in the Northwest area of Christchurch. Ages ranged from 6.5 years in Standard 1 to 13.0 years in Form 2 and a wide ability range was covered. Details of each class composition are given in Table 4.

Table 4

Description of Subjects in the Main Test Sample

Group	Class Level	Age Range	Male	Female	Total
1	Std.1	6.5-7.5	5	8	13
2	Std.1	7.0-8.0	15	4	19
3	Std.1 Std.2	7.0-8.0 8.0-9.0	3 9	17 6	20 15
4	Std.2	8.0-9.0	15	18	33
5	Std.3	9.0-10.5	19	13	32
6	Std.3 Std.4	9.0-10.0 10.5-11.0	8 7	9 6	17 13
7	Std.4	9.0-11.0	18	17	35
8	Form 1	10.0-12.0	8	18	26
9	Form 1 Form 2	11.0-12.0 12.0-13.0	4 5	5 8	9 13
10	Form 2	12.0-13.0	11	20	31
Total Subjects		6.5-13.0	127	149	276
	Std. 1	6.5-8.0	23	29	52
	Std. 2	8.0-9.0	24	24	48
	Std. 3	9.0-10.5	27	22	49
	Std. 4	9.0-11.0	25	23	48
	Form 1	10.0-12.0	12	23	35
	Form 2	12.0-13.0	16	28	44
Total Subjects		6.5-13.0	127	149	276

All 276 pupils from Standard 1 to Form 2 were involved in testing and 24 of them, four at each class level, were withdrawn for individual interviews. Prior to interviewing, the four subjects from each class level were selected by their teachers to match one of the following categories:

good mathematical and good language skills,
good mathematical and poor language skills,
poor mathematical and poor language skills,
poor mathematical and good language skills.

Teacher judgements were made on the basis of P.A.T. results, class tests and observation of behaviour in classroom activities. A balance between the number of boys and girls was taken into consideration. Finally two boys and two girls from Form 1 and Form 2 , three boys and one girl from Standard 3 and Standard 4, and one boy and three girls from Standard 1 and Standard 2 were chosen making a total of 12 boys and 12 girls. One Standard 2 girl had to be replaced by a boy as she was absent at the time of interviewing so that in the final sample there were 13 boys and 11 girls. The category represented by each student was not known to the interviewer until the completion of the trial programme.

4.2 Materials

The materials used in the test series are described in detail in the section on test development and included the following tests:

I. Counting Sequence
II. Counting/Numeration
III .Pattern Recognition and Continuation
IV. Short Term Memory
V. M-Space (Working Memory Capacity)
VII. Addition/Subtraction Problems
VIII.Long Term Memory/Prior Knowledge

Interview Outline

Ideally the interviewer would be able to obtain information about subjects' prior knowledge of number through open ended questions like,"Tell me about numbers." However subjects may be reluctant to talk freely or have difficulty in opening up a new line of discussion so a list of key questions to be covered in each interview was prepared. Discussion of these points would ensure subjects covered all aspects of their existing knowledge about number and provide information about their attitudes and abilities in other subjects. The questions were:

Why have numbers?
How are numbers used?
Which methods are useful in working with numbers?
What are numbers?
Where are numbers used?
When are numbers used?
Who uses numbers?
Which subjects do you like/dislike at school?
What sort of books do you read?
Why do we learn mathematics?

4.3 Procedure

Testing and interviewing was conducted over two consecutive weeks late in July. A programme was drawn up so that testing didnot interfere with other school activities or occur on days when normal school routine was disturbed by events such as a school production being performed at that time. Finally three days in each week were timetabled and the individual interviews arranged around the class tests. A resource room was used for interviewing subjects.

The prepared series of tests was administered to subjects from ten classes in their own classroom during the morning, one class before the morning tea interval and another class following the morning tea interval. A brief introduction to the test outlined the purpose of testing (to help the researcher understand how children learn about numbers) before distributing answer papers and beginning to read the instructions. Subjects wrote their name, age to the nearest half year and class level at the top of their answer sheets. All questions were read from the instruction sheets and time for recording answers measured on a watch with a second hand. In this way the researcher was able to handle the instructions, monitor time keeping and observe subject behaviour.

Eight of the ten class teachers stayed in the room while testing was conducted. The class teachers were able to help with monitoring the way instructions were followed, for example writing answers in the correct space at the beginning of the test. This was particularly useful for the Standard 1 subjects who had little experience of this type of recording answers on a separate sheet. Standard 2 to Form 2 subjects had experience of formal testing when they completed the Progressive Achievement Tests earlier in the year.

Testing time varied from 45 minutes with Form 1 and 2 subjects to 60 minutes for Standard 1 subjects. More time was spent on introducing the test and making sure instructions were understood and followed carefully with the younger subjects. During Test F, recording digits in the reverse order, a small number of subjects were seen writing from right to left so that they didnot have to reverse the order of digits heard. They were reminded to begin with the last digit heard, writing that digit next to the question number. No further problems were encountered in test administration.

Those subjects who appeared to have written from right to left on Test F questions had their answers marked incorrect. This decision was based on examination of the spacing used. Those six subjects who had left large spaces next to question numbers 61-70 or who had irregular spaces between digits, with those next to the question numbers being small, were thought to have completed this test incorrectly.

The researcher recorded correct answers on one answer sheet and this was used as a marking key. All answer sheets were marked twice, once by a seventh form high school student who was investigating teacher errors in marking as part of seventh form statistics project and once by the researcher. No errors in marking were found.

Alternative answers recorded on all tests except memory test were collated and analysed for common errors. This had the further advantage of providing another check on accuracy of marking as each item response was examined and any 'missed' correct answers could be quickly highlighted.

Finally all sub-test scores and common errors were collated and analysed for similarities and differences in performance for subjects at each class level, age groups and in the whole sample of 276 subjects.

CHAPTER 5

RESULTS AND DISCUSSION

Overview

The results are given in three chapters. This chapter looks at the results of the test series, Chapter 6 looks at test results for the 24 subjects who were interviewed and Chapter 7 discusses some of the main points that emerged in the interviews.

Counting, Memory and Problem Solving Test Series

The marking of answer sheets showed that difficulty of items at the five levels outlined in the method section was appropriate on nine of the eleven tests. Subjects who did not complete Level 1 and 2 items failed to complete higher level items and those subjects who completed Level 4 and 5 items had also mastered lower level items.

Adjustment to the pattern recognition Test E resulted in the Level 4 and 5 items exchanging levels and in fact this was the original order in the trial tests. Level 4 and 5 items exchanged level in Test B as many subjects who completed Level 5 items had failed on Level 4 items while subjects who completed Level 4 also completed Level 5. In the trial tests less able subjects had not attempted these items and other subjects completed both levels successfully so the comparative difficulty of items at each level was not questioned. Blank spaces were left where answers were not known by subjects in all classes suggesting that subjects answered as much as they could but did not make blind guesses.

Alternative answers were therefore likely to be based on reasoning and were analysed in an attempt to understand thinking processes used on all but the three memory tests where recall of digits was required.

5.1 Age, Class Group and Tests Administered

Correlation coefficients for individual tests and subject age and class group and between addition and subtraction Tests J and K and other tests in the series are given in Table 5.

Table 5

Correlation Coefficients for the Series of Tests

Correlation	Test A	Test B	Test C	Test D	Test E	Test F	Test G	Test H	Test I	Test J	Test K
Age	0.49	0.49	0.54	0.61	0.67	0.46	0.45	0.42	0.45	0.72	0.64
Class	0.45	0.45	0.52	0.60	0.70	0.48	0.49	0.47	0.46	0.69	0.64
Addition	0.58	0.64	0.76	0.74	0.68	0.49	0.54	0.46	0.68		0.76
Subtraction	0.55	0.59	0.69	0.72	0.72	0.46	0.57	0.46	0.70	0.76	

Test correlation with age and class group show very similar results for all eleven tests. Both age and class group were considered as school mathematics programmes are planned according to the class group which means subjects who have been promoted to a class group at younger or older ages than most subjects in that class will have experienced different instructional programmes than their age peers. As both correlation coefficients are similar placement of subjects has not influenced their performance.

All test show a positive correlation with age and class group and all coefficients, ranging from 0.42 to 0.76, are highly significant at the 0.01 level on a one tailed test of significance (Popham,1967 p396). Tests with lower age and class group correlations include the counting sequence Tests A and B, Counting/Numeration Test C, Memory Tests F, G and H, and Prior knowledge Test I. The addition and Subtraction tests J and K, Counting/Numeration Test D (subtract 10) and the pattern recognition Test E show a greater correlation with both age and class group.

A positive correlation between addition/subtraction and the other tests also exists and again the correlation coefficients are significantly high with a greatest figure of 0.76 between addition and subtraction Tests J and K and addition and counting/numeration Test C. Tests with relatively low correlations (<0.65) include the counting sequence Tests A and

B, and the memory Tests F, G and H. The other tests, counting/numeration, pattern recognition, prior knowledge an addition/subtraction have correlations between 0.68 and 0.76. Two of the memory tests , recalling digits (Test F) and counted totals (Test H) in the correct order have the lowest correlation (<0.5) with the addition and subtraction tests.

These results indicate that performance on the whole series of test shows an improvement with increase in age and class group. Correlations were positive suggesting a significant relationship with age, class group and ability to solve addition and subtraction examples. The memory test showed the relatively weakest correlation with ability to solve addition and subtraction examples and the aspects of memory tested here do not appear to be strong factors in solving arithmetic problems. Counting/Numeration, pattern recognition and prior knowledge appeared to be more closely related to problem solving ability.

These findings do not support the theory proposed by Case (1978, 1982) where mathematical development is explained in terms of increasing mental space (M-Space). Test H was included as a test of M-Space and this test showed the lowest correlation with addition and subtraction tests. Means and standard deviations for the series of eleven tests are given in Table 6.

Results reported for each class level and the whole sample of 276 subjects have been given to the nearest 0.5 level as this is consistent with the accuracy of marking used in indicating the type of item that subjects can or cannot complete. For example on Test A a mean level score of 4 indicates that subjects can name the next counting number after one, two or three digit numbers but experience difficulty with level 5 items like 209 .

Research on children's counting has shown this type of item to present problems as children continue the established pattern of the decades where the ones digit increases to 9 and then an increase in the tens digit occurs. When faced with this type of item subjects will increase the hundreds digit giving the next counting number as 300 rather than the correct response of 210.

All tests show an increase in mean level score with class ranking but levels achieved vary on the different tests. The standard deviation for Tests C, D, E, F, G, H, I, and K is one level so a majority of subjects score within one level of the mean level score and 95% of subjects within two levels each side of the mean level score on these tests. On Tests A, B and J involving naming counting numbers and addition examples a standard deviation of half a level means 66.7% of subjects scored within half a level and 95% within one level each side of the mean level score. Hence scores were not widely spread across individual tests.

Table 6

A Summary of Results for the Series of Tests

Class Group		Test A	Test B	Test C	Test D	Test E	Test F	Test G	Test H	Test I	Test J	Test K
Std. 1	Mean	4	3.5	2.5	2	1	2.5	2	1	2.5	2.5	1.5
	Std.dev.	1	1.5	1.5	1.5	1	1.5	1	1	1	1	1
Std. 2	Mean	4.5	4.5	4	3.5	1.5	3	2	1	3	3.5	2.5
	Std.dev.	1	0.5	1	1.5	1.5	1	1	1.5	1	1	1.5
Std. 3	Mean	5	5	4.5	4	2	3	2.5	1	3	4	3
	Std.dev.	<0.5	1	0.5	1	1.5	1	1	1	1	1	1
Std. 4	Mean	5	5	4.5	4.5	3.5	3.5	3	1.5	3.5	4.5	3.5
	Std.dev.	<0.5	<0.5	0.5	1	1	1	1	1	1	0.5	1
Form 1	Mean	5	5	4.5	4.5	4	4	3	2	3.5	4.5	4
	Std.dev.	<0.5	<0.5	0.5	0.5	1	1	1	1.5	1	0.5	1
Form 2	Mean	5	5	5	5	4	4	4	3.5	4	5	4.5
	Std.dev.	<0.5	<0.5	0.5	<0.5	1	1	1	1	1	<0.5	1
Total Sample	Mean	4.5	4.5	4	4	2.5	3	2.5	1.5	3	4	3
	Std.dev.	0.5	0.5	1	1	1	1	1	1	1	0.5	1

Counting tests A, B, C, D, and addition examples in Test J show overall good performance at Level 4 or 5 while Test H (recalling counted totals) was poorly done by a majority of subjects with a mean level score of 1.5 and memory tests F and G, pattern recognition Test E, writing number equations Test I and subtraction examples in Test K presented some difficulty with items at higher levels, that is, above Level 3.

5.2 Counting Sequence Tests A and B

On the counting tests A and B naming the number after or before a given number subjects' mean score was Level 4.5. A level score of 4.5 or 5 indicates that subjects successfully attempted all types of item in this subtest with at least one of the two items at Level 5 correct. This means that a majority of subjects from the Standard 1 to Form 2 sample are familiar with the standard counting sequence to 999, naming numbers in the correct order and understanding its structure so that they are able to generate any number in that sequence without counting from the beginning.

Results for the different class levels on Tests A and B show a mean score of Level 5 for classes above Standard 2 on both naming numbers after and before a given number while Standard 2 subjects score at Level 4.5 on these two tests and Standard 1 subjects at Level 4 on naming the next number and 3.5 on naming the previous number. An analysis of the answers given for individual items showed that it was mainly younger subjects who have difficulty with Level 4 and 5 items which involve three digit numbers and in particular the following type of examples:

Q7. Name the number after 200.....Level 4

Q9. Name the number after 209.....Level 5

Q18.Name the number before 800.....Level 5

For the first example Q7 the common alternative answer given by 18 Standard 1 and 5 Standard 2 subjects was 300. Once a hundred is reached counting then proceeds in hundred for these young subjects. A different approach was shown by the 6 Standard 1 and 6 Standard 2 subjects who correctly answered item Q7 but for Q9 also answered 300 where the hundreds rather than the tens digit has been increased by one. This follows the pattern of the decades but here as there are no tens the hundreds digit is increased by one.

Subjects had little difficulty naming the number before a number like 210 but did have difficulty with the third type of example given, Q18, where they gave 700 or 709 as the number before 800. Subjects who thought 300 followed 200 counted back in this manner and thought 700 came before 800 while those subjects who thought 300 followed 209 counted back from 800 to 709.

By Standard 3 the standard counting sequence to 999 is well established and the very small number of errors made show no consistent pattern. Standard 1 and 2 subjects had no difficulty with numbers below 100 and attempted to generate numbers in the counting sequence above 100 but made consistent errors in applying their understanding of the counting system. Subjects who counted in hundreds above 100 repeated the one to a hundred pattern of the first 100 counting numbers while subjects who went to the next hundred after 209 followed the pattern of the decades where after -9 the next decade begins but here there are no decades so the hundreds are increased instead.

These type of counting procedures were observed in the study by de Vere (1989) with Standard 2 subjects and were used by subjects who experienced difficulty with mathematics at school. Other research (Clements,1984;Fuson et al.,1982;Siegler and Robinson,1982)has found numbers like 209 to be a common stopping point where the standard counting sequence breaks down.

5.3 Counting/Numeration Tests C and D

All subjects showed a mean score of Level 4 and a standard deviation of one level on both Test C and D . A score at this level indicates that a majority of the subjects from Standard 1 to Form 2 were able to add and subtract ten from two or three digit numbers but had difficulty with Level 5 items. On Test C these items involved adding ten to numbers such as 294 and on Test D difficult items involved subtracting ten from numbers such as 407. Both of these examples involve a transition across a hundred.

Results for each class group show mean level scores for subjects above Standard 2 were 4.5 or 5 on Test C (adding ten to a given number) while Level 4 was the mean score for Standard 2 subjects and Level 2.5 for Standard 1 subjects. Test D (subtracting ten) showed a similar pattern of results but Standard 1, 2, and 3 subjects scored at a mean level 0.5 below their scores on the add ten test. The higher standard deviation recorded for Standard 1 and 2 subjects illustrates the wider spread of scores for younger subjects in the sample.

A level score of 3.5, 4, or 5 indicates that subjects are able to add or subtract to two digits numbers but experience some difficulty with three digit numbers. Items that caused some difficulty included the following Level 4 and 5 items:

Q29. Add ten to 294.........
Q37. Subtract ten from 700..
Q39. Subtract ten from 407..

An analysis of answers given for these items showed that for Q29 an answer of 394 was given by seven Standard 3 to Form 2 subjects and an answer of 307 for Q39 by 18 subjects across all class groups although 10 of the 18 subjects came from the Standard 1 and 2 classes. Incorrect responses to both these items indicate an increase/ decrease of one digit in the hundreds rather than the tens digit. Ten subjects recorded 700 subtract ten as 600 again reducing the hundreds rather than tens digit in the number. This type of error was made by Standard 1, 2, and 3 subjects.

As subjects gave answers of 394 for Q29 and 307 for Q39 they must be using a procedure where they adjust the relevant digit rather than counting on or back. Counting procedures would have given the correct answer of 304 for Q29 because the difficult stopping point of 309 is not reached and subjects who count in hundreds after 100 would have counted to 700 by counting on ten and no subject recorded this answer.

Some subjects gave a different type of answer for Q29 and Q39. Two subjects, one in Standard 3 and one in Form 2, recorded 306 as the answer for 294 add ten and six Form 1 and 2 subjects gave 393 as the answer for 407 subtract ten. In these examples subjects have counted to the nearest ten and added or subtracted that number from the nearest ten in the following manner:

294+[ ]=400......	407-[ ]=400
4+6=10 so 294+6=400	7-7=0 so 407-7=400
400+6=406	400-7=393

Subjects interviewed demonstrated the procedure that these subjects are attempting to use and it involves renaming ten so that this makes the calculation easier in the following manner:

294+10=[ ]	407-10=[ ]
294+6=300	407-7=400
6+4=10 so 300+4=304	7+3=10 so 400-3=397

Here subjects' knowledge of numbers are being applied in a problem soving situation and the error made demonstrates the type of reasoning possibly used by other subjects who recorded the correct answers for these items.

Subjects who made errors on adding and subtracting ten from a given number were not the same subjects who made errors on the two counting sequence Tests A and B. All but two subjects who made errors in the counting sequence tests did not attempt to add or subtract ten from three digit numbers. The two subjects who thought 700 subtract 10 was 600 made the type of error in the counting sequence tests where 209 was followed by 300. The other subjects who made errors when adding or subtracting ten had successfully completed items at all levels on the counting sequence tests.

This difference in performance on the two types of test suggests that subjects are not using counting procedures to add or subtract ten from a number. Subjects appear to use their knowledge of place value structure in completing these items but some subjects make errors with three digit numbers that have no tens adjusting the hundreds digit instead. If these subjects, who demonstrated the ability to generate numbers at any point in the standard counting sequence up to 999, had counted on or back ten numbers they would have arrived at the correct answer. Instead they attempted to use a more efficient procedure that is successful on all other three digit numbers such as those in items Q25, Q26, Q35 and Q36.

5.4 Pattern Recognition Test E

The mean score of Level 2.5 on the pattern recognition test is lower than the scores on the counting sequence and counting/numeration tests. A score of Level 2.5 means that the majority of subjects in the sample could recognise and continue repeating patterns such as 4,7,4,7 and familiar counting patterns with two, five or ten as the common difference between members but had difficulty with less familiar patterns with a common difference of three, four or eleven used in Level 4 and 5 items.

Mean scores for the different class groups show an increase from Level 1 for Standard 1 subjects to Level 4 for the Form 2 subjects. Many subjects, particularly the younger class groups left blank spaces on the answer sheet for items they couldnot complete on this test . Standard 1 subjects continued repeating patterns and addition of ten in the pattern 17,27,37,47.. but made few attempts at the more difficult items in Levels 3,4 and 5. Standard 2 subjects also recognised the pattern of adding eleven in item Q45 while Standard 4 to Form 2 subjects were successful with all but Level 5 items where the digits in the numbers formed patterns, the first/second digits in each number formed an ascending pattern while the second/first digits in each number formed a descending pattern.

There were three types of inappropriate methods of continuing a pattern. The first method involved adding one irrespective of the actual pattern in each item for example:

Q41. 4-7-4-7......8-9	Q43. 59-57-55-53..54-55
Q42. 17-27-37-47..48-49	Q48. 91-82-73-64..65-66
Q49. 2-5-8-11.....12-13	Q50. 98-94-90-86..87-88

The second method involved adding ten to the last number heard and since the sample pattern at the beginning of the test involved adding ten (10-20-30-40..50-60) these subjects may have applied this rule to all items. Evidence that this may not be the case is provided by answers given for the individual items . Unlike the previous method there was some variation in the way subjects answered different items as shown in the following examples provided by one subject:

Q41. 4-7-4-7......8-9	Q42. 9-1-9-1......10-11
Q43. 17-27-37-47..	Q44. 59-57-55-53..54-55
Q45. 11-22-33-44..54-64	Q46. 50-45-40-35..45-55
Q47. 98-87-76-65..75-85	Q48. 19-28-37-46..56-66
Q49. 2-5-8-11.....12-13	Q50. 98-94-90-86..96-106

This Standard 1 subject has added one to patterns that begin with single digit numbers and added ten to patterns with two digit numbers. The item left blank was the one item where adding ten would have given the correct answer but it is impossible to tell why this item is blank.It appears likely that the presence of two digit numbers is followed by a response to add on tens just as subjects in the study by de Vere (1989) counted on in tens first when faced with problems involving two digit numbers. This method of counting tens and then adjusting the ones is also outlined in research conducted by Resnick.

The third method of continuing a pattern recognised only a part of the pattern as shown in theses examples:

Q41. 4-7-4-7......5-8	Q42. 9-1-9-1......10-2
Q42. 17-27-37-47..57-58	Q44. 59-57-55-53..52-51
Q47. 98-87-76-65..64-63	Q46. 50-45-40-35..25-15
Q47. 98-87-76-65..55-45	Q48. 19-28-37-46..57-68

Q49. 2-5-8-11.....13-15   Q50. 96-94-90-86..76-66

In Q41 and Q42 the repeating pattern has been continued but one has been added to each number in the pattern. The common difference of ten has been added to the last number in the given sequence for Q43 but then one is added to this number instead of maintaining the common difference. The decreasing nature of Q44, Q46, Q47 and Q50 has been retained but one or ten has been subtracted as a common difference regardless of the actual pattern. An error in the common difference has occurred in Q49 adding two instead of three and in Q48 both digits have been increased by one when the second digit should be reduced by one in each subsequent term.

This last error was the most common for Form 2 subjects,five subjects responding in this way, although it was noted for one or two subjects at all other class levels. The greatest number of errors were made by Standard 1 and 2 subjects with few errors by the older subjects. Table 7 shows the percentage of subjects in each class group who made one of the three types of error described above.

The distribution of alternative answers for items given in Table 7 shows that it is the Standard 1 subjects in particular who made these types of error. The percentage of Standard 1 subjects who gave the correct answer only exceeds those who gave one of these three types of error on the first three items. More offered an alternative answer on the remaining seven items and the data in Table 7 does not include the percentage of subjects who made no response or gave a different type of answer, such as only recording the next number instead of the following two numbers.

A greater percentage of Standard 2 subjects gave one of the three alternative types of answer than the correct answer on three items Q46, Q48 and Q50. Overall they answered more items correctly and fewer items with one of the alternative answers than did the Standard 1 subjects. The percentage of subjects in Standard 3, 4, Form 1 and 2 recording a correct answer is greater than those offering an alternative for all ten items. The alternative answers are still given for all items by Standard 3 subjects but the percentage is less than for the younger subjects. Standard 4, Form 1 and 2 subjects have lower

Table 7

Percentage of Subjects who made Common Errors on Test E Items

Class Group	Answer Category	Q41	Q42	Q43	Q44	Q45	Q46	Q47	Q48	Q49	Q50
		Item Number									
	a	30	22	35	5	12	9	2	0	5	0
Std.1	b	12	14	11	16	18	11	7	9	21	9
	c	0	0	0	0	5	0	4	4	2	4
	d	4	7	4	9	0	0	4	4	4	2
Total	%bcd	16	21	18	26	18	11	14	16	28	14
	a	35	40	54	6	31	15	10	4	13	2
Std.2	b	8	8	6	2	10	2	2	4	6	6
	c	0	0	0	2	6	2	2	2	0	2
	d	0	6	2	2	0	19	4	2	6	2
Total	%bcd	8	15	8	6	17	23	8	8	13	10
	a	59	55	67	27	47	39	25	20	18	10
Std.3	b	6	2	2	4	2	2	2	2	4	0
	c	0	0	0	6	2	2	0	0	0	0
	d	0	4	0	8	0	8	6	4	10	6
Total	%bcd	6	6	2	18	4	12	8	6	14	6
	a	90	90	84	55	78	63	41	18	59	47
Std.4	b	0	0	0	0	0	0	0	0	2	0
	c	0	0	0	0	2	0	0	0	0	0
	d	0	0	0	4	0	0	2	4	0	2
Total	%bcd	0	0	0	4	2	0	12	4	2	2
	a	83	77	91	66	80	74	46	31	49	49
Form 1	b	0	0	0	0	0	0	0	0	0	0
	c	0	0	0	0	0	0	0	0	0	0
	d	0	0	3	11	0	3	3	3	0	0
Total	%bcd	0	0	3	11	0	3	3	3	0	0
	a	77	80	82	61	80	73	48	25	61	57
Form 2	b	0	0	0	0	0	2	2	2	2	2
	c	0	0	0	0	0	0	0	0	0	0
	d	0	0	0	0	0	5	2	11	2	0
Total	%bcd	0	0	0	0	0	6	5	14	5	2
	a	67	65	78	41	58	49	32	18	36	30
All	b	5	5	4	4	6	3	2	3	7	3
	c	0	0	0	1	2	<1	1	1	<1	1
	d	1	3	1	6	0	6	4	5	4	2
Total	%bcd	6	8	5	10	8	10	7	9	11	6

Category a-correct answer.
Category b-subjects add 1 to the last number heard.
Category c-subjects add 10 to the last number heard.
Category d-subjects complete part of the pattern.

Total %bcd=% of subjects who gave a category b,c or d answer.
Number of subjects at each class level:

Std.1	Std.2	Std.3	Std.4	Form 1	Form 2	Total
52	48	48	49	35	44	276

percentages recording one of the alternative answers and some items have no alternative answers given (Q41, Q42 and Q45 for all three class groups). These older subjects make mostly the type of error where part of the pattern is continued whereas the younger Standard 1 and 2 subjects make more of the error where they add one to the previous number. Addition of one or ten rarely occurs with the three senior classes but all class groups have subjects who partly continue a pattern.

Those errors made, adding one, adding ten or only retaining part of a pattern suggest that the counting procedures used in problem solving are being applied here by this group of subjects mainly from Standard 1 and 2 classes. The standard counting sequence is followed irrespective of the actual pattern and two digit numbers appear to trigger adding on a ten.

5.5 Memory Tests

A mean score of Level 3 on the digit span Test F and Level 2.5 on digits backwards Test G indicate that a majority of the subjects in the sample were able to recall five digits and four digits in reverse order. Individual class scores show an improvement with class level. Standard 1 subjects scored a mean of Level 2.5 on Test F and Level 2 on Test G indicating a recall of four digits and three digits in reverse order. Form 2 subjects scored a mean of Level 4 on both tests indicating a recall of six digits and five digits in reverse order.

Test H, a test of M-Space or mental space where totals have to counted and then recalled in the correct order proved to be the most difficult of the test series with a mean score of Level 1.5 for the whole sample. This score indicates that subjects were able to count and recall three totals in the correct order. Individual class mean scores show that Form 2 subjects performed at a higher level than other class groups recalling four totals in the correct order. Since subjects had performed at low levels on this test a further small study was thought necessary to investigate this further and discussion of these three memory test is continued later (see p 49).

5.6 Prior Knowledge Test I

Subjects were asked to write as many equations as possible in five minutes on this test and a variety of examples were provided by all age groups. Form 1 and 2 subjects included fractions, decimals, percentages, negative numbers, powers and square roots that younger subjects had not experienced. All class groups included multiplication examples and some division examples. Interest was not in the different operations or kinds of numbers used but in the difficulty level of the examples given in terms of using prior knowledge. In the following discussion only addition and subtraction of whole numbers is included although there are parallel multiplication/division examples and examples with other kinds of number such as fractional or decimal numbers.

Table 6 (p37) shows the mean score for the whole sample was Level 3 on this test with a standard deviation of one level. A score at this level indicates that a majority of the subjects were able to record equations involving doubles for example, 4+4=8, addition and subtraction facts to 18 including a change of order for the addends in addition examples like 4+3=7 and 3+4=7, and two digit addition/subtraction.

Class group mean scores show an increase from Standard 1, Level 2.5 to Level 4 for Form 2 subjects. Level 4 includes equations where subjects have demonstrated use of generalisations such as the effect of adding or subtracting zero, reversibility of operations. Equations given by younger subjects are based on recall of known facts and some calculation whereas the use of generalisations by the three older class groups demonstrates a more efficient use of existing knowledge which can be applied to many different numbers eliminating the need to store vast quantities of individual facts.

If subjects can only recall doubles, for example 2+2=4 then this is the only type of fact that they can use in problem solving without resort to some kind of counting procedure. On the other hand if subjects can recall rules rather than facts these may be applied to many situations as needed. Similarly recall of patterns or clusters of related facts (Level 5) indicate an efficient method of storing much data that is readily accessible by entry to the pattern or cluster.

5.7 Addition and Subtraction examples Test J and K

Table 6 (p37) contains the level scores for Test J, addition problems, and Test K, subtraction problems. Each item contained one missing number but from different parts of the equations.(A list of items is given on page 23).

A mean score of Level 4 for addition and 3.5 for subtraction items was obtained by the whole sample. A score of Level 4 indicates an ability to successfully equations with an unknown addend or sum,for example:

Q81. 3+4=[]...7

Q85. 9+[]=12..3

Q87. []+8=15..7

Level 3 on subtraction items includes missing addends but does not include equations where an initial sum is the unknown ,for example :

Q97. []-6=7..13

This type of subtraction problem requires an understanding of the reversibility of the operation 6+7=[] to find the solution. Examples at lower levels may be solved directly by using counting procedures in the following manner:

Q95. 15-[]=9

a) Count back from 15 to 9..

15-1, 14(1), 13(2), 12(3), 11(4), 10(5), 9(6).

b) Count on from 9 to 15

The equation may be changed to the more familiar form of 15-9=[] but if this is done with the numbers in the Level 4 example an error occurs...7-6=[] because the sum has been replaced by an addend.

Class group mean scores show an increase for both tests from Level 2.5 for Standard 1 subjects to Level 5 for Form 2 subjects on addition and Level 1.5 to Level 4.5 on subtraction for the same subjects. Standard 4, Form 1 and 2 subjects were successful on all types of addition item but only Form 2 subjects achieved a mean score at a level (4.5) where the most difficult items are included. This type of item involves an understanding of the equivalence of the two sides of an equation as shown in these examples:

Q90. 6+8=5+[ ]   Q100. [ ]-6=5+4

A heavier load is placed on memory in retaining more bits of data and working out intermediate steps in the calculation. The sum for one side of the equation is needed in order to find the missing number or equal adjustments to the addends in Q90 may be made by reasoning 5 is one less than 6 so the other addend must be one more than eight in order to retain the equivalence of the two operations.

An analysis of the incorrect answers for each item on these two test showed subjects had given an answer one away from the correct answer on all items for example Q43. 4+9=12 which may be the result of counting on incorrectly:

9 (1) 10(2) 11(3) 12(4)..the answer is 12.

Different kinds of common error were made on Level 5 items Q89 and Q90 on the addition test and Level 4 and 5 items Q97, Q98, Q99, and Q100 on the subtraction test as the following examples show:

Q89. 2+9=4+[ ]......15 for 7 subjects
.......5 for 6 subjects
Q90. 6+8=5+[ ]......14 for 15 subjects
.......8 for 11 subjects
Q97. [ ]-6=7 ....1 for 10 subjects
.......7 for 4 subjects
Q98. [ ]-3=8 ....5 for 11 subjects
.......8 for 5 subjects
Q99. [ ]-7=1+7.......1 for 10 subjects
.......8 for 68 subjects
Q100.[ ]-6=5+4.......3 for 10 subjects
.......9 for 52 subjects

For Q89 seven subjects added all the numbers to get an answer of 15 while six subjects ignored the 2 and answered the problem 4+[ ]=9 to get an answer of 5. An answer of 14 for Q90 indicates that the 15 subjects who gave this answer solved the problem 6+8=[ ] and ignored the 5. The Level 5 subtraction items Q99 and Q100 show that the subtraction operation has been overlooked by those ten subjects who retained the equivalence of the equations through addition in answering 1 for Q99 and 3 for Q100. Answers of 8 for Q99 and 9 for Q100 only solve one side of each equation. The answers given for the Level 4 items Q97 and Q98 suggest subjects have changed the equations to the more familiar form , 7-6=[] and 8-5=[].

These errors demonstrate how some subjects attempt to follow well known procedures developed to cope with the familiar form of written school arithmetic, that is add or subtract two numbers to get an answer.

5.8 Memory Tests and Time Interval

Performance was poor for many subjects on the M-Space Test H in the test series. The follow up series of tests investigated the effect of the time interval between members of a sequence and whole sequences in tests that require recall of digits or counted totals. Tests were based on the three memory tests in the main study, Tests F, G and H, with differences in the time interval between members of an item in each test. The new tests included:

Test S. Recall a series of digits in the given order.
(One second between each digit)

Test T. Recall a series of digits in the given order.
(No interval between digits, total time <2 sec.).

Test U. Recall a series of digits in reverse order.)
(One second between each digit.)

Test V. Recall a series of digits in reverse order.)
(No interval between digits, total time >2 sec.)

Test W. Recall counted totals of a series of letter names in the given order.
(One second between each letter name/group.)

Test X. Recall counted totals of a series of letter names in the given order.
(One second between each letter name and two seconds between each letter group.)

Test Y. Recall counted totals of a series of letter names in the given order.
(No interval between letter names or groups.)

Test Z. Recall counted totals of a series of letter names in the given order.
(No interval between each letter name and two seconds between each letter group.)

All tests included ten items with two items at each one of five levels of increasing difficulty as more digits or letters were added to the sequence to be recalled. The results for this series of tests are given in Table 8.

Table 8

Summary of Results for the Time Interval Tests

Group		Test S	Test T	Test U	Test V	Test W	Test X	Test Y	Test Z
Whole Sample	Mean	2.5	3.5	1.5	1.5	1.5	0.5	1.5	2
	Std.dev.	1	1	1	1	1	0.5	1.5	1.5
Low Recall	Mean	2	2.5	0.5	0.5	0.5	<0.5	<0.5	0.5
	Std.dev.	0.5	1	0.5	0.5	0.5	0.5	0.5	0.5
High Recall	Mean	3	4	2	2	2	0.5	2.5	2.5
	Std.dev.	0.5	0.5	1.5	1	1	0.5	1.5	1.5

Test S was identical to Test F in the main test series where subjects had to recall digits in the correct order. A one second interval between presentation of each digit occurred. Test T involved one change to this procedure, digits were read with no interval between each digit so that the whole sequence was read in less than two seconds. The mean score for the whole sample of 20 subjects was Level 2.5 on Test S with one second between digits, and Level 3.5 on Test T, with no interval between digits. This shows an improvement of one level when there is no interval between digits. Tests U and V required digits to be recalled in reverse order. Test U was similar to Test G in the main series and had one second interval between digits. There was no interval between digits in Test V where the whole sequence was read in less than two seconds. The mean scores for the whole sample show no difference for these two tests.

The M-Space test,Test W was not identical to the main series Test H where subjects counted sounds heard on a tape cassette recorder. Here the test administrator read a sequence of letters with one second between each letter, for example, FFF SS OO and subjects had to count and recall totals in the correct order. The sample in this follow up experiment consisted of subjects from Group P in the trial series. A comparison between mean scores in these tests and the trial tests show Level 2.5 for both Tests F and S and Level 1.5 and 2 on Test T and G. The mean score for Group P in the trial tests on Test F was Level 0.5 while the mean score for the subjects in this experiment was Level 1.5. The mean scores on the recall of digits show a similar performance on both tests but the M-Space test mean score has shown an improvement of one level as a result of changes in presentation.

Removing the time interval between letters in Test X resulted in a very low mean score of Level 0.5. Test Y and Z were the same as Tests W and X but two seconds time interval was given between groups of identical letters. Test Y with one second between weach letter and two seconds between groups of letters resulted in a mean score the same as Test W where all intervals were one second long. Reading each group of identical letters as a whole unit with no interval between each letter but two seconds between groups of letters in Test Z gave a mean score of Level 2 similar to Tests W and Y.

The results for these eight tests showed that the time interval did have an effect on some tests. No interval between members of sequence increased recall of digits in Test T but lowered the totals recalled in M-Space Test X. A one or two second interval between groups of letters in Tests W, Y and Z only made a difference when there were no intervals between individual letters as in Test X and Z. An interval between groups of letters but not individual letters in Test Z had the same mean score as Test W where a one second interval occurred between both letters and groups of letter. It appears that a time interval either between letters or groups of letter is important in succeeding on this type of test.

An examination of answers recorded across all eight tests showed that some subjects appeared to score at high levels on several of the tests while other subjects performed at low levels on all tests. Subjects were divided into two equal groups of ten according to their score on Test S, recall of digits in the correct order.Those with a Level score of Level 2 or lower were assigned to the low recall group. A score of Level 2 indicates the ability to recall four digits in the correct order. Subjects in the high recall group, had a score above Level 2 which indicates a recall of five or more digits.

The results for the two ability groups in Table 8 show very low scores of Level 0.5 or less on all but Tests S and T which involve recall of digits. Hearing the digits as a sequence with no interval between digits shows a small mean score improvement of 0.5 to Level 2.5 for this group indicating recall of five digits. The high recall group increased their mean score by one level on this test to Level 4 indicating a recall of six digits at this level. Mean scores on the tests where digits are recalled in reverse order Test U and V show the same mean score, Level 2 and M-Space tests show a pattern of results like those for the whole sample where no interval reduces the mean score to Level 0.5 and tests with an interval between letters or groups of letter have mean scores of Level 2 or 2.5. This level indicates a recall of four counted totals. Level 0.5 scored by the low recall group only indicates recall of two counted totals.

Not all subjects utilise the interval given. The low recall group demonstrated the ability to recall digits in the order heard,though at a low level, but performed poorly on all other tests. The high recall group appear to use the interval in some way that helps their recall. At the time of testing subjects were observed using their fingers to keep track of data and quietly repeating data to themselves so this type of strategy may be used by the high recall group but without further investigation exactly what this group of subjects do in those critical one or two seconds cannot be described.

This experiment had only involved 20 subjects from a Standard 1,2,3 class grouping. The results had revealed a difference for subjects of high and low recall ability. School records showed that those subjects in the low recall group had scored at class percentiles below C.P.R. 20 on Progressive achievement Test in Mathematics administered earlier in the year.Subjects in the high recall group had higher percentile ranking on this test. Those subjects with low ability in mathematics appeared to make no use of time interval in the tests given although the critical factor in processing data was not apparent from these tests it appeared that some aspect of memory was important because the high recall subjects used the time interval to process data. The scores for memory tests in the main test series were re examined to see if further useful data could be obtained from this larger sample.

Four groupings of subjects were formed on the basis of performance on the three memory tests with a high score being above Level 2.5 and a low score Level 2 or below:

46 Group HHH ..High Test F and High Test G and High Test H
64 Group HHL...High Test F and High Test G but Low Test H
29 Group HLL...High Test F but Low Test G and Low Test H

78 Group LLL...Low Test F and Low Test G and Low Test H

This gives a total of 217 subjects. The remaining 59 subjects scored at Level 2 and 3 on the three tests but difference in scores on tests was small so these subjects may be considered to perform at average levels on all tests.

Group HHH and HHL included five of the six interview subjects, with good mathematics and language ability, three with good mathematics/poor language ability and one with poor mathematics / good language ability. Group HLL included one subject with good mathematics/poor language ability, three with poor mathematics/good language ability and one with poor mathematics/poor language subject. Group LLL included four subjects with poor mathematics/poor language ability and one with poor mathematics/good language ability.

The break down of subjects in each group offers some support for the findings of the small experiment designed to investigate the effect of interval given between members of an item and recall of that item. Subjects who performed at low levels on Test F, recall of digits, scored at low levels on Tests G and H. This group consisted of 78 of the 276 subjects in the sample and only 29 subjects who scored at a high level on Test F scored at low levels on both other memory tests. Only one subject had a low score on Test F, Level 0 and a high score on Test G, Level 4. Subjects who have difficulty with recall of data are likely to have difficulty working with data if they cannot remember that data and so this finding is not unexpected but subjects in this category tend to those with poor mathematical ability. As suggested in the time interval tests the critical factor appears to be in processing data rather than recalling it.

CHAPTER 6

INTERVIEW SUBJECTS' TEST RESULTS AND GROUP DATA

All interviews were successful in drawing out relevant material about subjects' understanding and experience of number. Transcripts of the interviews were analysed for points raised by more than one subject and this data, obtained from the 24 individual interviews, is summarised according to class grouping in Table 9 and according to ability groups in Table 10 .

Class grouping shows that the Standard 1 subjects produced the lowest number of points (61/200) across all categories but particularly on the 'How numbers are used' category (5/32) . Form 1 and 2 subjects show a higher number of responses than the other class levels on the 'What numbers are' category (13/24) .Across all question categories Standard 2 and 3 subjects show a similar rate of response (41/200 and 42/200) that is greater than the youngest subjects but less than the rate of response for the three older class level groups, Standard 4, Form 1 and 2 (52/200, 59/200, 55/200).

Further analysis of the data based on points common to at least three of the four subjects at each class level highlights similarities and differences betweeen the groups. Standard 1 subjects note that numbers are needed for counting quantities, for order and location of items and they recalled seeing numbers in books, on letter boxes/classrooms and for sizes/grades/scores of items. They use fingers and/or work on paper or in their heads and use numbers for counting. All of these common points illustrate an emphasis on the standard number sequence and counting at a concrete level in their understanding of number.

Standard 2 and 3 pupils saw numbers as necessary for order/location and identification of items and recalled seeing numbers in books ,on letterboxes/classrooms, signs and transport using them for dates, time, distance/speed and grade/score/sizes of items. Counting and comparision were examples of how numbers are manipulated either on paper or working in heads. Counting is still important at this level but awareness of equivalence is appearing.

Table 9

A Summary of Interview Data for Different Class Levels

Why

Subject	1	2	3	4	5	6	7
S1A	x	x	x	x			
S1B			x		x		
S1C	x		x	x			
S1D				x	x		
S1 Total	2	1	3	3	2	0	0
S2A	x		x	x	x		
S2B							
S2C	x		x	x	x		
S2D				x	x	x	
S2 Total	2	0	2	3	3	1	0
S3A	x		x	x			
S3B						x	
S3C	x		x		x		
S3D	x		x	x	x	x	
S3 Total	3	0	3	2	2	2	0
S4A	x	x	x	x	x	x	x
S4B	x	x	x	x	x		
S4C	x		x	x	x		
S4D	x	x	x	x	x		
S4 Total	4	3	4	4	4	1	1
F1A			x	x		x	x
F1B	x	x	x	x		x	x
F1C	x		x	x			x
F1D			x	x			x
F1 Total	2	1	4	4	0	2	4
F2A	x		x	x		x	x
F2B	x	x	x	x	x	x	x
F2C	x		x	x			x
F2D			x	x		x	
F2 Total	3	1	4	4	1	3	3

How

Subject	1	2	3	4	5	6	7	8
S1A					x		x	
S1B	x							
S1C					x		x	
S1D							x	
S1 Total	1	0	0	0	2	0	3	
S2A	x	x	x	x	x		x	x
S2B	x	x	x	x	x			
S2C					x		x	
S2D					x			
S2 Total	2	2	2	2	4	0	2	1
S3A	x	x	x	x	x	x	x	x
S3B	x	x	x		x		x	x
S3C					x		x	
S3D								
S3 Total	2	2	2	1	3	1	3	2
S4A	x	x	x	x	x	x	x	x
S4B	x	x	x	x	x		x	x
S4C	x	x	x	x			x	x
S4D								
S4 Total	3	3	3	3	2	1	3	3
F1A	x	x	x	x	x	x	x	x
F1B	x	x	x	x	x	x	x	x
F1C	x	x	x		x		x	
F1D	x	x						
F1 Total	4	4	3	2	3	2	3	2
F2A	x	x	x	x	x		x	
F2B	x	x	x	x			x	
F2C	x	x	x	x				
F2D	x						x	
F2 Total	4	3	3	3	1		3	0

Which

Subject	1	2	3	4	5
S1A	x	x	x	x	
S1B	x	x	x	x	x
S1C	x	x	x	x	
S1D		x	x		
S1 Total	3	4	4	3	1
S2A		x	x	x	x
S2B		x	x		
S2C	x	x	x	x	
S2D	x	x	x		
S2 Total	2	4	4	2	1
S3A	x	x	x		
S3B	x	x	x		
S3C	x	x	x	x	
S3D	x	x	x		
S3 Total	4	4	4	1	0
S4A		x	x		x
S4B		x	x		
S4C	x	x	x		x
S4D	x	x	x	x	x
S4 Total	2	4	4	1	3
F1A	x	x	x	x	x
F1B	x	x	x	x	
F1C	x	x	x		x
F1D	x	x	x		
F1 Total	4	4	4	2	2
F2A		x	x		
F2B	x	x	x		
F2C	x	x	x		
F2D	x	x	x	x	x
F2 Total	3	4	4	1	1

What

Subject	1	2	3	4	5	6
S1A	x			x	x	
S1B						
S1C				x		x
S1D						
S1 Total	1	0	0	2	1	1
S2A			x		x	x
S2B				x	x	x
S2C						x
S2D						
S2 Total	0	0	1	1	2	3
S3A	x			x	x	x
S3B						
S3C						
S3D						
S3 Total	1	0	0	1	1	1
S4A				x		x
S4B	x	x		x	x	
S4C		x				
S4D						
S4 Total	1	2	0	2	1	1
F1A	x	x	x	x	x	x
F1B				x	x	x
F1C			x			
F1D	x	x	x			
F1 Total	2	2	3	2	2	2
F2A	x	x	x	x	x	x
F2B		x	x		x	x
F2C		x				
F2D		x	x			
F2 Total	1	4	3	1	2	@

Where

Subject	1	2	3	4	5	6	7	8	9
S1A			x		x				
S1B					x				
S1C			x		x				
S1D			x	x	x				x
S1 Total	0	0	3	1	4	0	0	0	1
S2A	x	x	x	x	x			x	x
S2B			x		x		x	x	
S2C			x				x	x	x
S2D			x	x	x				x
S2 Total	1	1	4	2	3	0	2	3	3
S3A	x		x	x	x				x
S3B		x		x	x			x	
S3C				x	x		x	x	x
S3D			x		x		x		x
S3 Total	1	1	2	3	4	0	2	2	3
S4A	x		x	x	x				x
S4B	x		x		x	x	x	x	
S4C					x		x		
S4D	x					x	x		x
S4 Total	3	0	2	1	3	2	3	1	2
F1A	x	x	x	x	x	x	x	x	x
F1B		x	x	x	x		x		
F1C	x				x	x	x		
F1D				x		x	x		x
F1 Total	2	2	2	3	3	3	4	1	2
F2A				x	x	x	x	x	x
F2B					x	x			x
F2C		x	x		x		x	x	x
F2D		x	x	x	x		x		
F2 Total	0	2	2	2	4	2	3	2	3

When

Subject	1	2	3	4	5	6	7	8
S1A								
S1B			x				x	
S1C	x			x			x	x
S1D	x	x	x				x	
S1 Total	2	1	2	1	0	0	3	1
S2A	x	x			x			
S2B					x		x	
S2C	x	x			x		x	x
S2D	x	x	x				x	
S2 Total	3	3	1	0	3	0	3	1
S3A		x		x		x	x	
S3B	x	x		x		x		x
S3C		x	x	x		x		x
S3D								x
S3 Total	1	3	1	3	0	3	1	3
S4A		x	x		x		x	
S4B	x		x					
S4C	x		x			x	x	x
S4D	x		x					x
S4 Total	0	1	4	0	1	1	2	2
F1A	x	x	x	x	x	x	x	x
F1B	x		x		x			x
F1C	x	x	x					
F1D	x	x	x				x	
F1 Total	4	3	4	1	2	1	2	2
F2A	x	x	x	x	x	x	x	x
F2B	x		x		x		x	
F2C	x		x	x	x			
F2D	x							
F2 Total	4	1	3	2	3	1	2	1

Who

Subject	1	2	3	4	5	6	7	
S1A	x	x						
S1B	x							
S1C	x							
S1D			x					
S1 Total	3	1	1	0	0	0	0	
S2A	x		x			x		
S2B								
S2C								
S2D								
S2 Total	1	0	1	0	0	0	1	
S3A		x						
S3B	x		x					
S3C				x	x	x	x	
S3D								
S3 Total	1	1	1	1	1	1	1	(
S4A		x			x		x	
S4B								
S4C					x	x		
S4D			x			x		
S4 Total	0	1	1	0	2	2	1	(
F1A		x				x	x	
F1B			x			x	x	
F1C								
F1D			x					
F1 Total	0	1	2	0	0	2	2	(
F2A				x	x	x	x	
F2B				x		x		
F2C	x			x	x	x		
F2D								
F2 Total	1	0	0	3	2	3	1	0

Why
1. To get a job.
2. Future education.
3. Counting .
4. Order/Location .
5. Identification .
6. Measurement.
7. In everyday life .

How
1. Add
2. Subtract
3. Multiply
4. Divide.
5. Order
6. Reverse
7. Count f+b
8. Multiples

Which
1. Fingers
2. In head
3. On paper
4. Ruler
5. Calculator

What
1. Numerals
2. Odd/Even
3. Digits
4. Place value
5. Infinite
6. Equivalence

Where
1. Everywhere
2. Phone
3. House nos.
4. Clock
5. Books
6. Shops
7. Money
8. Signs
9. Transport

When
1. Date
2. Time
3. Price/paying
4. Temperature
5. Dist./speed
6. Weight
7. Score/grade
8. Games

Who
1. Teacher
2. Student
3. Parent
4. Shopkeeper
5. Banker
6. Workers
7. Everyone

x-Subject includes this point in discussion

The four operations, addition, subtraction, multiplication and division are added to counting as a way of manipulating numbers by the three older class level subjects who also mention seeing numbers on money when it is used for prices and payment of items as well as for dates. They regard numbers as necessary for order/location and
quantifying items. Although counting is still used operations are now explained in meaningful terms and it is interesting to note the emergence of money and prices in their common experience of numbers at this level.

Addition, subtraction and to a less extent 'times' were mentioned by younger subjects but their explanation of these operations consisted of counting sets to find an answer rather than performing actual operations on numbers. Their experience relates to use of the standard number sequence for counting items or using the sequence to place/find items in order in situations like when finding a page number or delivering a letter. Older children who handle money describe operations.

Data summarised for the different ability groups show Group A (good mathematics and language) produced the greatest response rate over all question categories (186/200) while Group D (poor mathematics and languge) produced the lowest overall response rate (99/200). Group B (good mathematics and poor language) and Group C (Poor mathematics and good language) produced similar response rates (136/200 and !33/200).

Further analysis of the data focusing on points raised by five of the six subjects in each ability group highlight similarities and differences between the four groups. All groups noted numbers were needed for order/location and were found in books. Counting was mentioned as the method of manipulating numbers although only 3/6 Group D included this point.

Group A subjects explained how numbers were used in the four operations and counting and were able to describe what numbers are in terms of the infinite nature of counting, the place value structure of the number system and meaning of equals. Calculations were performed in heads or on paper and their experience and application of number was wide including books, clocks, transport or just everywhere for measuring time, distance/speed and grades. People in work situations were

Table 10

A Summary of Interview Data For Different Ability Groups

Subject	Why							How								Which					What					
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	8	1	2	3	4	5	1	2	3	4	5	6
S1A	x	x	x	x								x		x		x	x	x	x		x			x	x	
S2A	x		x	x	x			x	x	x	x	x		x	x		x	x	x	x			x		x	x
S3A	x		x	x				x	x	x	x	x	x	x	x	x	x	x			x			x	x	x
S4A	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x		x				x		x
F1A			x	x		x	x	x	x	x	x		x		x	x	x	x	x	x	x	x	x	x	x	x
F2A	x		x	x		x	x	x	x	x	x	x					x	x			x	x	x	x	x	x
A.Total	5	2	6	6	2	3	3	5	5	5	5	5	3	4	4	3	6	6	3	3	4	2	3	5	5	5
S1B			x		x			x								x	x	x	x	x						
S2B								x	x	x	x	x					x	x						x	x	x
S3B						x		x	x	x		x		x	x	x	x	x								
S4B	x	x	x	x	x			x	x	x	x	x		x			x	x			x	x		x	x	
F1B	x	x	x	x		x	x	x	x	x	x	x	x	x	x	x	x	x	x					x	x	x
F2B	x	x	x	x	x	x	x	x	x	x	x			x		x	x	x				x	x		x	x
B.Total	3	3	4	3	3	3	2	6	5	5	4	4	1	4	2	4	6	6	2	1	1	2	1	3	4	3
S1C	x		x	x								x		x		x	x	x	x					x		x
S2C	x		x	x	x							x		x		x	x	x	x							x
S3C	x		x		x							x		x		x	x	x	x							
S4C	x		x	x	x			x	x	x	x			x	x	x	x	x		x		x				
F1C	x		x	x			x	x	x	x	x	x		x		x	x	x		x			x			
F2C	x		x	x			x	x	x	x	x			x		x	x	x				x				
C.Total	6	0	6	5	3	0	2	3	3	3	3	4	0	6	1	6	6	6	3	2	0	2	1	1	0	2
S1D				x	x									x			x	x								
S2D				x	x	x								x		x	x	x								
S3D	x		x	x	x	x										x	x	x								
S4D	x	x	x	x	x											x	x	x	x	x						
F1D			x	x		x	x	x	x							x	x	x			x	x	x			
F2D			x	x		x		x						x		x	x	x	x	x		x	x			
D.Total	2	1	4	6	4	4	1	2	1	0	0	0	0	3	0	5	6	6	2	2	1	2	2	0	0	0

Subject	Where									When								Who						
	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7
S1A			x		x													x	x					
S2A	x	x	x	x	x			x	x	x	x			x				x		x			x	
S3A	x			x	x				x		x		x		x	x			x				x	
S4A	x		x	x	x				x		x	x		x		x			x			x		x
F1A	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x				x	x
F2A	x			x	x	x	x	x	x	x	x	x	x	x	x	x	x				x	x	x	x
A.Total	5	2	4	5	6	2	2	3	5	3	5	3	3	4	3	4	2	2	4	1	1	2	4	3
S1B					x		x					x				x		x						
S2B			x		x		x	x						x		x								
S3B		x		x	x			x		x	x		x		x		x	x		x				
S4B	x		x		x	x	x	x		x		x												
F1B		x	x	x	x		x			x		x		x			x			x			x	x
F2B					x	x			x	x		x		x			x				x		x	
B.Total	1	2	3	2	6	2	4	3	1	4	1	4	1	3	1	2	3	2	0	2	1	0	2	1
S1C			x		x					x			x			x	x	x						
S2C			x			x	x	x	x	x	x			x		x	x							
S3C				x	x		x	x	x		x	x	x		x		x				x	x	x	x
S4C					x		x			x		x			x	x	x					x	x	
F1C	x				x	x	x			x	x	x												
F2C		x	x		x		x	x	x	x		x	x	x										
C.Total	1	1	3	1	5	2	5	3	3	5	3	4	3	2	2	3	4	1	0	0	1	2	2	1
S1D			x	x	x				x	x	x	x				x				x				
S2D			x	x	x				x	x	x	x				x								
S3D			x		x		x		x								x							
S4D	x					x	x		x	x		x					x			x			x	
F1D				x		x	x		x	x	x	x				x				x				
F2D		x	x	x	x		x			x														
D.Total	1	1	4	4	4	2	4	0	5	5	3	4	0	0	0	3	2	0	0	3	0	0	1	0

Why
1. To get a job.
2. Future education.
3. Counting .
4. Order/Location .
5. Identification .
6. Measurement.
7. In everyday life .

How
1. Add
2. Subtract
3. Multiply
4. Divide.
5. Order
6. Reverse
7. Count f+b
8. Multiples

Which
1. Fingers
2. In head
3. On paper
4. Ruler
5. Calculator

What
1. Numerals
2. Odd/Even
3. Digits
4. Place value
5. Infinite
6. Equivalence

Where
1. Everywhere
2. Phone
3. House nos.
4. Clock
5. Books
6. Shops
7. Money
8. Signs
9. Transport

When
1. Date
2. Time
3. Price/paying
4. Temperature
5. Dist./speed
6. Weight
7. Score/grade
8. Games

Who
1. Teacher
2. Student
3. Parent
4. Shopkeeper
5. Banker
6. Workers
7. Everyone

x-subject includes this point in discussion mentioned as examples of people who use numbers. Numbers were needed for getting a job, quantifying items and order/location. Overall this group demonstrated a sound understanding of number based on wide experience.

Group B subjects explained number operations, counting and comparing items and some of these subjects described counting as infinite (4/6), the place value of number system and meaning of equals (3/6). They used paper or their heads for calculations and noted seeing numbers in books and on money. Numbers were used for prices and dates. Quantifying items was the reason given for having numbers by 4/6 subjects in this group. The understanding of number shown by this group was similar to that for Group A subjects but their description of where, when, why and who uses number was not so full as that given by the other group and may be explained by their inferior language skills in expressing their ideas.

Group C explained counting items and used fingers as well as paper or their heads for calculations. Numbers had been seen in books and on money and were used for dates, prices and in games. Getting a job, quantifying items and order/location of items were reasons given for having numbers. Standard 4, Form 1 and 2 subjects also explained operations whereas Standard 1,2 and 3 subjects talked about comparing numbers.

Group D subjects did not explain how numbers are used or what numbers are. They often counted items or used fingers to solve problems although they mentioned using their heads or paper. Books and dates were examples of where and when numbers are used and they are needed for order/location of items. Quantifying and identification of items were also noted by 4/6 subjects. Addition and subtraction were explained as counting sets of items to find the answer.

The two groups A and B with subjects who had good mathematical ability explained counting, operations and described numbers in terms of the standard counting sequence and the number system. They did not rely on concrete aids for calculation. Groups C and D with poor mathematical ability were dependent on counting for calculation and problem solving and frequently used objects or fingers to help them. The two groups with good language ability were able to express their ideas and saw numbers

Table 11

Test Series Results for Interview Subjects in Class Groups

Subject	Age	Test A	Test B	Test C	Test D	Test E	Test F	Test G	Test H	Test I	Test J	Test K
S1A	7.5	5	5	5	5	0.5	4.5	2.5	4.5	4	4.5	3
S2A	8.5	5	5	5	5	3	4.5	2.5	4.5	5	4	4
S3A	9.5	5	5	5	5	4.5	4.5	5	0.5	4	5	4.5
S4A	10	5	5	5	5	4.5	3.5	2.5	1.5	5	5	5
F1A	11.5	5	5	5	5	4.5	4.5	3	2	5	5	4.5
F2A	12.5	5	5	5	5	5	5	4.5	2.5	5	5	5
Group A	Mean	5	5	5	5	4	4.5	3.5	2.5	4.5	5	4.5
S1B	7.5	1	0.5	1.5	1	0.5	1.5	1	0	2	1.5	0.5
S2B	8.5	5	5	5	4.5	1.5	3.5	2	1.5	2	3.5	2.5
S3B	10	5	5	5	4.5	3.5	4	2.5	1.5	4	4	4
S4B	10	5	5	5	5	4	3.5	2.5	1	3	5	4.5
F1B	11.5	5	5	5	5	5	3.5	1.5	1.5	4	5	4
F2B	12.5	5	5	5	5	3.5	4.5	4	4	4	4.5	4
Group B	Mean	5	5	5	5	3.5	4	2.5	2	3	4.5	4
S1C	8	4	3.5	2.5	2	0	1	1.5	0.5	2	3.5	2.5
S2C	8.5	5	5	5	4.5	2.5	2	3.5	3.5	2	4	4
S3C	9	5	5	5	4	1.5	4.5	1.5	0.5	2	3.5	1.5
S4C	10.5	5	5	5	4.5	2.5	3.5	1.5	1.5	3	4.5	4.5
F1C	11.5	5	5	4	5	4.5	4.5	2	1	3	4.5	4
F2C	12.5	5	5	5	5	3.5	5	5	4.5	3	5	4.5
Group C	Mean	5	5	4.5	4	2.5	3.5	2.5	2	2.5	4	3.5
S1D	7.5	3	2.5	1.5	1	0	0	1.5	0	1	0.5	0.5
S2D	8	5	5	4.5	4.5	1	1.5	1	0	2	2.5	1
S3D	9.5	5	5	4.5	4.5	1.5	3.5	1	0	2	4	1.5
S4D	10.5	4.5	5	1	0	1	3.5	2	0.5	2	0.5	0.5
F1D	12	4	5	2.5	3.5	2	3.5	2.5	0.5	2	4	2
F2D	12	5	5	2	4.5	0	2.5	3	4.5	3	5	4
Group D	Mean	4.5	4.5	2.5	3	1	2.5	2	1	2	3	2

as necesssary for getting a job, quantifying items and order/location of items. The poor language groups B and D had the lowest response rate on the when question category and failed to show links between categories as they were unable to explain why numbers were needed in the places they mentioned seeing them or why people would need to know about numbers .

The scores on the series of tests for each of the 24 subjects interviewed are shown in Tables 11 and 12. Subjects are grouped according to class level in Table 11 and according to ability in Table 12 .

Looking across the whole series of tests the Standard 1 subjects show lower mean scores than other class groups but there is wide variation in individual scores on several of the tests. All four subjects scored at a very low level on the pattern recognition test, Test E, but subject S1A performed at a level more consistent with older subjects on the other tests.

When subjects were grouped according to ability, in Table 12, Group D showed the lowest level scores and Group A the highest level scores across all tests. Group B and C subjects show a similar pattern of performance. In discussion of individual sub-tests the Standard 1 (S1B, S1C, S1D) and Group D (S1D, S2D, S3D, S4D, F1D, F2D) subjects are discussed separately as their performance differs from the subjects in other groups.

Performance varied on the different sub-tests. The counting Tests A and B were mastered at all levels showing a good understanding of the standard number sequence and ability to generate numbers at different points in the sequence. The small number of subjects who had difficulty with this test S1B, S1C, S1S, S4D, F1D coped well with numbers less than one hundred but with numbers above one hundred they wrote the number they heard instead of the next number. One subject S1C gave 301 as the number following 200 and subject S1D gave 300 for the answer to this item. Both of these answers demonstrate addition of one to the hundreds, in the first example one is also added to the ones digit, and this illustrates misuse of a generalisation that the next counting number is obtained by adding one to a number.

Counting/numeration Tests C and D also show mastery of items at all levels for Group A subjects but S2B, S3B, S2C, S3C and S4C had difficulty on Level 5 items when subtraction across a hundred was required (e.g. subtract 10 from 605) and F1C had difficulty with addition across a hundred (e.g. add 10 to 691). Answers given illustrate two methods of dealing with this item. The most common alternative answer for this item was 791 instead of 701 resulting from the addition of a hundred instead of a ten suggesting the misuse of a generalisation that the digit is increased by one but here the hundreds not tens digit has been increased. The other alternative answers such as 302 or 303 for addition of 10 to 294 suggests that a counting on method is being used and small errors in keeping track of how many have been counted on have occurred.

The Standard 1 and Group D subjects had difficulty with numbers above one hundred on these two tests .Standard 1 subjects found both addition and subtraction of ten difficult but four Group D subjects scored at higher levels on these items. Answers given by F2D for the two sub-tests show higher levels

Table 12

Test Series Results for Interview Subjects in Ability Groups

Subject	Age	Test A	Test B	Test C	Test D	Test E	Test F	Test G	Test H	Test I	Test J	Test
S1A	7.5	5	5	5	5	0.5	4.5	2.5	4.5	4	4.5	3
S1B	7.5	1	0.5	1.5	1	0.5	1.5	1	0	2	1.5	0.5
S1C	8	4	3.5	2.5	2	0	1	1.5	0.5	2	3.5	2.5
S1D	7.5	3	2.5	1.5	1	0	0	1.5	0	1	0.5	0.5
Std.1	Mean	3	3	2.5	2	<0.5	2	2.5	1	2	2.5	1.5
S2A	8.5	5	5	5	5	3	4.5	2.5	4.5	5	4	4
S2B	8.5	5	5	5	4.5	1.5	3.5	2	1.5	2	3.5	2.5
S2C	8.5	5	5	5	4.5	2.5	2	3.5	3.5	2	4	4
S2D	8	5	5	4.5	4.5	1	1.5	1	0	2	2.5	1
Std.2	Mean	5	5	5	4.5	2	3	2	2.5	3	3.5	3
S3A	9.5	5	5	5	5	4.5	4.5	5	0.5	4	5	4.5
S3B	10	5	5	5	4.5	3.5	4	2.5	1.5	4	4	4
S3C	9	5	5	5	4	1.5	4.5	1.5	0.5	2	3.5	1.5
S3D	9.5	5	5	4.5	4.5	1.5	3.5	1	0	2	4	1.5
Std.3	Mean	5	5	5	4.5	3	4	2.5	0.5	3	4	3
S4A	10	5	5	5	5	4.5	3.5	2.5	1.5	5	5	5
S4B	10	5	5	5	5	4	3.5	2.5	1	3	5	4.5
S4C	10.5	5	5	5	4.5	2.5	3.5	1.5	1.5	3	4.5	4.5
S4D	10.5	4.5	5	1	0	1	3.5	2	0.5	2	0.5	0.5
Std.4	Mean	5	5	4	4	3	3.5	2	1	3	4	3.5
F1A	11.5	5	5	5	5	4.5	4.5	3	2	5	5	4.5
F1B	11.5	5	5	5	5	5	3.5	1.5	1.5	4	5	4
F1C	11.5	5	5	4	5	4.5	4.5	2	1	3	4.5	4
F1D	12	4	5	2.5	3.5	2	3.5	2.5	0.5	2	4	2
Form 1	Mean	5	5	4	4.5	4	4	4	1	3.5	4.5	3.5
F2A	12.5	5	5	5	5	5	5	4.5	2.5	5	5	5
F2B	12.5	5	5	5	5	3.5	4.5	4	4	4	4.5	4
F2C	12.5	5	5	5	5	3.5	5	5	4.5	3	5	4.5
F2D	12	5	5	2	4.5	0	2.5	3	4.5	3	5	4
Form 2	Mean	5	5	4	5	3	4	4	4	4	5	4.5

scored on the addition than subtraction items. For this subject answers given on addition items show addition of a hundred rather than a ten but counting errors occurred on subtract ten items. This subject appears to have used two different methods of processing data on the two tests attempting to use the structure of the number system for addition of ten but counting back on the Level 5 subtraction items. This may explain the lower levels scored on addition items.

Errors made by two other Group D, S2D and S3D, subjects who scored at Level 4.5 on the add/subtract ten sub-tests show counting errors, 303 instead of 304 for 294 add 10 and 25 for 16 add ten, indicating a preference for counting methods for these subjects. Results of the tests administered to all subjects showed that subjects who scored at Level 5 on counting sequence

Tests A and B had some difficulty with adding and subtracting ten when they tried to use the place value structure but adjusted the wrong digit. Although slower counting methods may be more accurate for subjects who have not yet completely mastered use of the structure underlying the number system.

The standard counting sequence itself may be easier to use in counting back rather than forward once numbers over one hundred are reached . When saying the next number after one hundred a common error is to go to 200, 300...and after 109 some children will begin to increase hundreds saying 109, 200,201, 202...209,300.... following the pattern of the decades where each decade includes 1 to 9 as the last digit and then a change in the tens digit. In moving from the known to the unknown children apply the rules that worked previously but when rules are misapplied the standard counting sequence breaks down.

Different stopping points seem to occur in counting forward and backwards as shown by the results of Test B, name the number before a given number. Naming the number before a whole number of hundreds was more difficult than naming the number before numbers like 209. When counting back from 10 the sequence 9,8,7... is well established and is readily applied to 210, 209.... When counting forward children applying well known generalisations will see 209..300 as logical in following the pattern of the decades but in counting back it seems logical to go from 210 to 209. Unlike counting forward where non standard names such as sixty ten may occur it is impossible to continue with a non standard name when counting back because children cannot go back beyond zero and so the need to do something different at this point is obvious.

Pattern recognition Test E shows scores at a very low level for all four Standard 1 subjects. S1D made no attempt at any item, S1C wrote the two numbers that followed the last number read out (e.g. 4,7,4,7....8,9) ,S1B completed only one of the two repeating pattern items (9,1,9,1...9,1) and S1A completed only one item involving the addition of ten to each previous member (17,27,37,47...57,67) . The four Standard 2 subjects scored low levels on this sub-test when compared with other class groups although S2A scored at level 3 finding difficulty with items that involved a decreasing number sequence. Group D subjects also found this sub-test difficult and only successfully completed the items at Levels 1 and 2 involving repeating patterns and addition of ten. Group A subjects with the exception of S1A and S2A coped well with items at all levels on this subtest.

Memory tests F, G and H show Form 2 subjects perform well on all three tests, while other class groups score at lower levels on recall of digits in reverse order and recall of counted totals in sequence although each group of four subjects includes a wide spread of level scores.

Test I required subjects to write their own equations. Those written by Group A subjects show clustering of equations and number patterns indicating their recall links ideas in some way. Group B examples include generalisations such as addition/ multiplication of one and zero. Groups C and D only gave examples at the levels where counting on or recall of known facts occurs. The different class groups all included a range of levels on this test.

Test J show all subjects except S1B, S1D, S2D and S4D score at Level 4 or 5 demonstrating a good ability to solve all types of addition examples. Level 5 examples such as 2+9=4+[] require the equivalence of the two sides of the equation to be appreciated. The most common alternative answer was 15 obtained by adding all the numbers together. Another alternative answer was 5 which may be obtained by ignoring the 2 and changing the problem to 4+[]=9.

Test K shows more variation among subjects with all Group A scoring at level 4 or 5 except S1A who scored at level 3. Group B and C subjects ,except S1B, S1C, S1C and S3C, and F2D also scored at level 4 or 5. The other five Group D subjects scored at low levels. Four subjects S1B, S3C,S1D,S2D, S4D scored at Level 1 which only involves simple equations with numbers up to ten. It is possible to use counting on fingers to answer this type of problem. Three subjects, S1C, S3D and F1D scored at level 2 which involves simple equations with numbers to 18 and these items may be answered by counting methods but counting on or back rather than counting all is needed when using fingers because the sum in these examples is above ten.

For one example at this level, 17-8=[] two of subjects gave 10 as the answer indicating a counting error when the subject has counted the starting number as their first count e.g. 17 (1) 16(2) 15(3)... 10(8) instead of 17-1, 16(1),15(2), 14(3)...9(8). The three subjects who scored at level 3; S1A, S2B and S3C, solved examples of the type 15-[]=9 again requiring counting back or counting on methods if counting procedures are used. Alternative answers for this item, 7 and 5, indicate counting errors have occurred. Starting to count back from 15 or on from 9 when subjects count the starting number as their first count results in an answer of 7 which is one too high.

Subjects who scored below level 4 generally did not attempt any answer for level 4 and 5 items. Subjects who did answer these items incorrectly gave 8 as the answer for []-7=1+7 and 9 as the answer for []-6=5+4. These answers may be obtained by completing 1+7=[] and 4+5=[] thus ignoring the other part of each equation. This type of reasoning illustrates a strong committment to the tradition form of equation a+b=[] when two numbers are added to give an answer and only one correct answer to a sum. In other words school taught procedures are strictly applied. A different approach used by F1B, F1C and F2D involved treating both sides of the equations as an addition resulting in answers of 1 for the first example and 3 for the second example. These subjects are attempting to maintain the equivalence of the two sides of the equation but have failed to take account of the operations involved.

Performance for the interview subjects was representative of the large scale test results for the whole sample of 276 subjects. Those subjects with good mathematical ability scored highly on the tests while subjects with poor mathematical ability appear to have relied on counting and use of procedural knowledge resulting in lower test scores.

CHAPTER 7

INTERVIEW DATA FOR INDIVIDUAL SUBJECTS

The analysis of common points covered by groups of subjects during the interviews revealed general insights into subjects' understanding of number and ways of processing information but a more thorough examination of the individual subject responses enables a deeper understanding to be extracted from the data.

It would be difficult to include the vast volume of information related by the 24 subjects but the main points of interest are grouped into a small number of main categories.

7.1 Description of Number

Each interview commenced with the request to ,"Tell me about numbers." Subjects responded in a variety of ways:

Int: Tell me about numbers.
F1A: Um.. A number is an idea that you form in your head and once you write it down on paper it's a numeral and a number is an idea and they're used in lots..virtually in all..lots of things in the world because um.. well it's one of the basic..the world gets along with numbers and they're used on clocks and they're on the clocks..1,2 up to 12 and you can tell the time by that and they're used in telling what the year is like 1989 and things like that and numbers can also be used for grades...If you're the best sometimes you're 1 and if you're not the best sometimes you're 5 or whatever but you can use them as grades. They can be used as Part 2 or Part 2, things like that..They're used for counting like from 1 right up to infinity and um.. they're used for adding and subtracting and multiplying and dividing and do I talk about fractions?
Int: If you want to.
F1A: O.K. and um..from 1, from 0 to 1 that's called fractions and they're just a bit of 1 and um..it's a little bit of 1 and then below 1 they're called decimals like... and what else?...
Int: Just think about it.
....

F1B: Well..um.. numbers is um.. well it's an idea more or less. You can think of any number..it's infinite, it goes on for ever and ever and um....counting numbers, it's um...to help, to do with maths and it's to help with placings etc. You could, um, like first, second, third in a running race. If you didn't have them then you couldn't, you could call it something else, it could be a different whatever (laughs) but that's the best way of judging them or putting them in order. Oh yes they're good for putting on order and like in..you do questions in your maths book or your exercise book.. or just putting them in order and um....I know ..if you didn't have any you would be a wee bit confused .. what to do for maths cos maths... numbers is the whole idea of it. If you didn't have numbers then I don't think, you couldn't do much for maths..It um..... umm.. that's about all I can think of.

Subjects F1A and F1B came from the same class and both explained the abstract nature of numbers, infinite nature of counting and order as used in placings and grades. These subjects were able to organise and explain their ideas with little prompting from the interviewer although F1A has more organisation in her explanation. Younger subjects made shorter responses:

Int: Tell me about numbers.
S2A: Umm..they all equal up to something..umm..they stop at zero and then they all just start over again and then you get and umm... it goes on for ever and ever and.... they go in order, they're in order..umm....there's bigger groups and then.....you always start with one and you finish up with a made up one but..umm..numbers are made up for a special thing. There's numbers all through the year um...numbers are used for objects....

S2B: Oh well there's a whole lot of them..everlasting.. and ..each time you add another zero on it it goes up like, um, um it goes up,like ,like tens and then hundreds, then thousands, then millions..
Int: Tell me about adding a zero and they're everlasting.
S2B: umm.

S3A: You can count them ..you can keep counting for ever and you can add them together to get a new one and write them down..you can use them for grading things ...you can write them down and add them and you can add any ones together.
Int: How do you use numbers for grading things?
S3A: You can say 1 is the easiest, 2 is a bit harder and 3 is the hardest..and you could go the other way and 3 is the easiest and 1 is the hardest number..Or you could have more than 3.

These three subjects referred to counting as everlasting but responses are brief and required questioning to draw out their ideas. The structure of the number system was referred to by S2B and an older subject, F2A, provides a comparison with an older subject following up a point briefly discussed:

Int: Tell me about 9.
F2A: Well it's not a prime cos it's divisible by 3, 1 and 9, it's odd..umm..you can times it, divide it, and subtract it and add it um..it's a whole number... um... It can go in like 987 and it would take the place of 9 hundreds....

Int: Oh I tell you what else you mentioned ..decimal system. Perhaps you could explain that a bit more..How our decimal system works.
F2A: Well that's, it's from a hundred like, it's easy to work with money sort of like 75 cents is three quarters. You can write it as point 75.
Int: Yes and how does it all work, our decimal system. Can you explain how the whole thing..
F2A: Oh from base 10 which is a base 10 and ..um..Oh things like that...I'm not sure..
Int: Well if you were to write a number down that might help explain it.
F2A: Oh say 67 that'd be 6 tens plus 7 ones . that's um, you put it in sort of columns so that's one (points to 7) that's ten (points to 6) and hundreds and that's how you work out the numbers. Say 687 would be 7 ones, 8 tens and 6 hundreds like that.
Int: O.K. Can you put any more numbers this way?
F2A: Oh no not if you want, you can add them that way, not really unless it goes like a point.
Int: Yes then you can. Now what's that point for?
F2A: Oh that's to show that there are decimal numbers..to show that..um..
Int: O.K. you write some more down this way and see if you can explain what they are when the numbers get that side.

F2A: Oh this has become the first one like that would be eight tenths, then seven hundredths, and then thousandths...
Int: You said it's base 10 ..Can you think of any other things or explain what base is?
F2A: Um well computers work in base one don't they. Just count up 1,2,3,4..Um..
Int: Is there any other base system?
F2A: There's eight.
Int: Right and what happens in base eight?
F2A: Oh well like um, in our system 10 is sort of where it changes from, into two digits. You see in base eight, eights the highest one sort of you go 1,2,3,4,5,6,7,8 and then it would be like 10.

The level of explanation including different bases demonstrates how older subjects expanded on concepts that all subjects were able to include.

The examples included above were typical of those for Group A and B subjects (good mathematics ability) outlining points about the number system in counting and relations between numbers in comparing or operating on numbers. A different picture of numbers was described by Group C and D subjects including these examples:

F1C: Numbers are digits which you count with. They've been used for a long, long time. Well you can get different sorts of numbers, like in different languages. They have, they have, different well counting and things and yonks and yonks and yonks ago they probably counted stones and well numbers are used for adding up and subtracting and etcetera.
Int: Well what's etcetera..You'd better say it.
F1C: Um..times..um..divide and that's about it and um.. counting is used for lots of sorts of machines like to help count..like calculators and um, oh those telethings and well numbers are kind of quite important cos if you didn't have them you'd probably.. in lots of your books you wouldn't know how many there were and, um, numbers are used for new dates, counting how many things there are and just counting all sorts of things.

Int: Tell me about numbers.
S3D: Numbers are made for counting.umm.To help other people.
Int: How do they help other people?
S3D: Um..Um.. So..So they know how many things they've got...money..umm..I think lots of people use them.. um..for work.um.. for when they need quotes.
Int: Quotes?
S3D: Yes.
Int: Yes What other kind of jobs do you need them for?
S3D: You need them for a bank and..er..brick..um..kind of a brick..builder.
Int: Yes. Why does a builder need them?
S3D: Cos he has to know how ... he has to know how long ..um the wood has to be.
Int: Yes and what does your dad do for a job?
S3D: Um..he's a um..measures blinds.
Int: Oh so that's how you know about quotes.

Int: Tell me about numbers.
S2D: Well I know you can count them up and you learn your tables with them and you say how old you are ...and you count up all the numbers..
Int: Aha anything else you can think of
S2D: No.

Counting features prominently among these responses as the use for numbers at school or at work. Subjects' own experience of numbers is drawn on in describing how numbers are used. No actual description of counting numbers or the number system is included. Two of the Standard 1 subjects merely reeled off a list of facts:

Int: Think hard and tell me anything about numbers..
S1B: Ten hundred equals a thousand.50+50=100, four 50s equals 200, 1+2=3, 4+4=8, 8+8=16, 10+10=20, 30+30=60, 40+40=80, 21+2=23, 80+10=90..
Int: And how do you know all these things?
S1B: (laughs) We do maths..and I just remember all of them.

Int: Tell me about numbers. It can be anything you can think of.
S1C: 2 and 2 equals 5, 5 and 5 equals 10, 6 and 6 equals 12, 1 is greater than 2, mmmm, 2 is greater than...1..This used to be my brother's class... 3 and 3 equals 6, 10 plus 1 equals 11,..20 and 20 are 40, 3 tens are 30, 4 tens are 40....
Int: Well what else do you know about numbers, different things?
S1C: 1 is less than 2, 2 is greater than 1.....

Subject S1B behaved like Group D subjects in many ways. He was the most difficult subject to interview as his attention continually wandered and he needed constant probing to draw out information. His knowledge of numbers appeared to be confined to recalling facts. Both S1B and S1C listed mainly doubles such as 20+20=40 and these type of facts are the earliest recalled. S1C included errors, 2+2=5 and 1 is greater than 2 which was corrected. Her attention was broken by an unrelated reference to the room formerly being her brother's classroom demonstrating a break in her line of thought.

Unlike Group A and B those subjects with poor mathematical ability related very little understanding of what numbers are but described their experience in counting in different situations. References to order by F1C were for filing information or finding a location. Group A and B subjects expanded their description of counting on further questioning:

Int: What other things can you do with numbers?..You've told me counting, you've..perhaps you want to say a bit more about counting..What's infinity? You said you can go up to infinity.
F1A: Infinity is the last number in everything..it's just the last number..and it's every single whole number in the universe. That's the end of infinity and um.... and counting you can count in ones, which is just basic 1,2,3,4 and you can count in twos that's 2,4,6,8 and you can count in threes and and fours.. 3,6,9 and fours is 4,8,12,16 and so you can count in lots of things..um

Int: O.K. Now that's an excellent answer. Perhaps you could tell me a bit more about counting. You told me you could count forever. What else do you know about counting? How many ways can you count? What do you think?
S3A: You can say it and write them down.
Int: O.K. Well if I had a whole lot of stuff here. How would you count it?
S3A: Well you could put them in groups of 5 or 10 and then count the groups.

Int: Well why would that be better to do that?
S3A: Well otherwise it would be easy to get mixed up if you've got lots of things. If you've just got a little group then you can count them one at a time....And you can count them backwards...

Int: O.K. you've told me quite a bit there. Let's have a look at some of the things you've said..Right you said they're in order and you start with one. Why do they have to be in order?
S2A: Umm..because if they weren't in an order they'd go all funny places..9..7..3.4 .
Int: And would that matter?
S2A: No.
Int: Why do we have to have them in an order then?
S2A: So they rhyme
Int: Right..anything else?
S2A: Umm...no.
Int: O.K. then you said they stop at 9 and they start again. What do you mean by that?
S2A: Well they go 1,2,3..9 and then like they go 11,12,13.. and then after you've done all them they go back to 20, 21,22 and 23 and when that's all finished you back to 31 and start again.
Int: And well how does...Why do we have to do all that?
S2A: So you count.
Int: So you can count and why do you need to know how to count?
S2A: So then..to know how much objects there is.
Int: O.K. and you said there's bigger groups. What do you mean by bigger groups?
S2A: Oh..9 is bigger than 8....2 is bigger than 1..

Further questionning of other subjects gained little further information:

Int: Well see if you can tell me about counting numbers... Why do we count?
S2D: So we know how old we are.
Int: Can you think of any different things?
S2D: Trees, people, walls.
Int: Well how do you count?
S2D: Just speak.
Int: Well show me how you do it.
S2D: 1, 2,
Int: And what's next..what do you do after that?
S2D: Stop.
Int: When do you stop?
S2D: When you want to.
Int: So how far could you go?
S2D: Quite a wee way.
Int: Quite a way. How far can you count?
S2D: I don't know.
Int: You don't know ..What do you reckon?
S2D: About 200.

This subject has limited counting ability and demonstrates no understanding of the underlying structure.

7.2 Reason for Numbers

A variety of reasons were given for learning about numbers:

Int: Now you said numbers are the base of maths..so what's maths all about? What do you think?
F2A: Numbers...sort of like....
Int: That's a hard question. I wonder if you could think about that for a minute. It is about numbers but what's maths really all about ..thinking about numbers.
F2A: It's working out how, like problems, like you can cope with everyday life..like going to the shop you need to work out how much money you need. You need to have maths, know maths and in your job you might need maths to help you.
Int: What kind of jobs?

F2A: Oh an accountant, shopowner, nearly everything.
Int: So how important do you think learning maths at school is?
F2A: Oh yes, pretty important, yes.
Int: How would it compare with say learning to read?
F2A: Oh probably the same importance.

F1A: Numbers are used to show the price of things and you have the dollar sign to show prices and um.. money sort of comes with decimals..The dollar is the basic thing but less than a dollar that's sort of a cent and that's sort of a decimal cos that's a wee bit of a fraction and a wee bit of a whole number and like when you go to the grocery shop you've got $6 and things..$6.70 and say you know how much you're paying in money and um.. and the money's also in tens and hundreds and ones so that's pretty logical..it's pretty easy.. and..also to help you add and subtract and multiply and divide you've also got calculators.

F1C: Well we learn about numbers because they're very useful especially when you get older and,and you have to get a job cos if you didn't have numbers it would be just about impossible .
Int: Why would it be impossible?
F1C: Because well you couldn't keep track of things..cos when people keep files and notes and everything and they wouldn't know where to look it up maybe and they'd be there for a very, very long time looking for what they want.
Int: What else might you need to know them for?
F1C: Well numbers are mainly used for, well they're used for filing things to keep track of things, make it easier for people ...um.. they're used for..um..all sorts of things really, just for living.

F1D: Buying things, on calendars, birthdays,car registration
Int: So we've got buying things,car registration, calendars, birthdays..
F1D Amount of money.....to work things out with..
Int: Yes .What sort of things?
F1D: Like if you need to buy something you need to know the price of it..
Int: Yes. How would you work it out then?
F1D: Ah..do maths.
Int: Do you know how to?
F1D: I don't know how to do it.
Int: You're not very good..I bet you're better than you think.
F1D: No...I'm not sure. I'm not very good at my tables. I'm learning my times tables .

Int: And why do you think we have numbers?
F2C: Um...well just to help us learn..um...how many like how many books might be in that box there or how many people in this school or things like that...life generally..um..umm.
Int: What else are numbers used for?
F2C: Money (laughs) um...
Int: Now what about addition and subtraction?
F2C: Umm..yes I think they should be taught because like you can count, not being greedy, your money and see who's taken it or um, when a things lost you can tell if it's left or not if you know subtraction..um..and addition just to add money..accounts and things like that... Yes I think it's necessary but multiplication I think you could just use adding, just a lot of adding.

S3C: If we didn't have numbers we couldn't have money and we couldn't go to the moon.
Int: Why couldn't we go to the moon?
S3C: Cos ..um..to get to the moon you have to know your numbers because you have to be real good at maths.. cos.um..to go on the moon you have to..to be one of the people that.....and go to the moon you have to know your numbers good cos they've got maths computers.
S3C: And why do you have to know numbers to work with computers?
S3C: Oh because..so you're going to hit something you have to make certain calculations...how long to try and steer away.

Int: Well why do you learn the kind of maths you do at school. Why do you think you learn all of that?
S3C: Oh because it..it's as everyone has to work with numbers. Well not everyone-not people in the bush sometimes...oh yes not native people and you have to know your numbers when you grow up you get..if you didn't know your numbers you won't get a job..Right?

All of these subjects mention using numbers for a job and in particular refer to money.

Int: Well why do we have numbers?
S1C: So we can count.
Int: Yes and what do we use that for?
S1C: Count to work out sums.

Int: Why do you think we learn maths at school?
S1C: So you can, so you can, when you're out and about and you want a job they have to see your report and all that and they want to see how good you are and how..if it's some computer work you might want um..well they have to see mostly your your maths kind of thing
Int: Why would that be important?
S1C: So when you need, when you want a job and you hand in your report and they see how good it is and if it's not .. if it's bad and not all very good and all that they don't take it. So you need it for that and all..
Int: O.K. so they look at it and say that's good you're very good at maths. What are you going to do with the maths, all the things you've learned at school, all your maths what are you going to use it for?
S1C: You're going to use it for sometimes..
Int: Well what do you uses maths for?
S1C: To count..counting, learning..no not learning..
Int: You've spent all this time at school learning maths. What are you going to do with it when you've learned how to do it? You're going to use it some how. You're learning your times tables aren't you? Well what are you going to do with them when you know them?
S1C: Like if you're big, like a mummy, you can help your children do their homework and you might get some maths homework and you need it for that if you can remember it. You tell them how to do it and then they can do the rest.

Int: Why do we have numbers?
S1A: To learn maths.
Int: Yes and why do we need to do that?
S1A: Cos if you didn't know how to ..ummm..how to learn, know maths, you wouldn't know your tables and stuff.
Int: Why do you need to know your tables?
S1A: (Laughs) Oh..
Int: Why do you think you have to learn your tables?..Do you know all of yours?
S1A: I'm only up to my sevens.
Int: Well that's pretty good and why have you been learning all of them?
S1A: Because you will get tests. Like you could get umm..you get fast fifties and fast ones..There'd be real fast ones in about five seconds.
Int: Are you good at those?
S1A: We haven't had those yet, only the Standard 2 but I do have to get my tests right in maths.
Int: Well why do you have to learn them. What are you going to do with them when you know them all?
S1A: Umm...
Int: You haven't thought about that have you? What are you going to do with all those tables when you know them all?
S1A: Well if you be a teacher and you, and you know your tables well you could..umm you could learn them to other kids.
Int: Yes and who else might need to know them?
S1A: Ah..well big standard people.
Int: Yes and why would they need them?
S1A: ......mmmm...(shrugs)..

These younger subjects refer to school based activities such as tests and see no practical application except for teaching facts to other children in the future. The final example illustrates a common school activity with numbers, that is to find the answer.

Int: Why are we learning about numbers?
S1B: In case one day you were doing some stuff on a calculator and you didn't know what it equalled so you pushed the numbers and then you added it up and then you find the answer.

7.3 Estimation

Four subjects referred to using calculators and this was followed up by further questionning about being sure that the answer given was the correct one:

Int: Do you often work things out in your head?
S4A: Er..yes mainly. We use calculators at school about every three months to get us used to using calculators when we grow up. and when it just gives us a page of the newspaper which has all the car sales and we have to add up all the cars beginning with T...with the calculator and it's quite easy...Not easy some days because you..there's a big long sale and some are quite short and you have to push plus or something if you've got a big long number, keep pushing plus but I've done it before.
Int: How do you know if you've got the right answer on the calculator?
S4A: Um..well you can't really trust them. I just remember if it's 37, 37 plus 37 I just sort of do them, 70 or so and then he puts.. when it comes round on the calculator as 74 I say that's it.Your answers about 70.

Int: Yes.How do you know if you get the right answer on a calculator?
F1A: Um..Er..You've sort of got to estimate first.
Int: And do you do that alright?
F1A: Yes and you estimate it when you work it out on the calculator and it's got to be near what you've got.
Int: Well how do you estimate?
F1A: Well you just..the..er..um..first number, say if it's like 62x63. You'd take the first number like just put six sixes are 36 and you've sort of got..through 6.. You have to get round there.
Int: So what sort of answer do you expect for 62x63?
F1A: About 300 and ..360...no what was it?..62x63..No about 300 and..3 000, about 3 600...Oops.(laughs).
Int: What happened there?
F1A: I had a bit of a muck up.
Int: That's O.K.

Int: How do you know if you've got the right answer with a calculator?
F1C: I usually double check it to see if it's right but you never trust a calculator cos the battery might be running out.

Int: How do you like working things out at maths time?
S4C: By using my fingers, or when it's calculator time I press the buttons.
Int: Do you like calculator time?
S4C: Ahum..
Int: How do you know if you've got the right answer?
S4C: Cos the teacher writes it up on the board and you have to read it on the board.

The two Group A subjects describe how they estimate answers by rounding numbers and using known facts to check the answer is of the right order. The two Group C subjects make no attempt to estimate but offer double checking or waiting until the teacher writes the answer on the board as the way of ensuring your answer is correct. Using a calculator involves button pressing for these subjects whereas it is a more useful tool for the Group A subjects who by estimating demonstrate their understanding of the processes involved.

^7.4 Number Operations^

Although number operations were mentioned not all subjects were able to explain these operations in a meaningful way.

Int: Well tell me a bit more about some of the things you've already said....Well let's start with addition..What's addition?
F1A: Addition is when you've got one number and you add one or two more numbers to it and that's used in things like..um..say if you've got a set of something and then you get another set of something...Just count both sets and then add them together and see what you've got, the whole lot..um.. and that's really good and subtractions when you've got a number of anything, a number of things or something and they can, you get one number and then you subtract things from it, so you take things away from it and that's used, like you've got some fish and some of them die...You can find out how many fish you've got left and um.. subtractions when you've got like a number and instead of saying like if you want to multiply a 4, instead of saying 4+4+4+4 you just say 4 times 4 and that's much easier and you've got to know your tables at school up to, well to 10, but like I was taught to..most people go to 10 but I was taught to go to 12 ...
Int: You said subtraction but I don't think you meant that. What is it?
F1A: Multiplication (laughs).
Int: I thought you'd made a mistake.
F1A: I'm sorry.
Int: That's alright. Don't worry about it.
F1A: Oh it's multiplication and that's used when you've got..Oh that's used in the business world a lot.
Int: Why do you say that?
F1A: Because it's harder stuff..it's harder than addition and things like that ..and then there's division and that's the opposite to multiplication and if you've got like a..um.. a big number and you want to divide it then you half it or something. or quarter it or something..You just divide it by 4 or something or 2 and then you've got an answer...
Int: What does divide mean?
F1A: It means to um..well if you've got a whole number..it means to um.. it's virtually the opposite to multiplication. Multiplication is like 4 times 4.. division is 16 divided by 4 is 4 and so it's just undoing multiplication.....and um..

This example is typical of that given by the Group A and B subjects in the three older class groups. The four operations, addition, subtraction, multiplication and division are described in terms of joining and separating sets for addition and subtraction, repeated addition for multiplication and its reverse operation division is perceived as undoing multiplication. Counting is mentioned when joining sets as a method used for finding the sum. Examples from younger subjects describe the operations but do not include an explanation of the reverse operation.

Int: What happens when you add numbers together?
S3A: You get a new one which is both of them.
Int: Right. What else can you do with them except adding them.
S3A: You can subtract them.
Int: What happens when you do that?
S3A: When you get..what you you get left is the number you didn't take away.
Int: Is there anything else you can do.
S3A: You can times them and you can divide them.
Int: What's times ..when you do times.
S3A: Well say if you've got some paper and you want to divide it out equally you get the number of people and times it.
Int: O.K. So why are you doing when you do times?
S3A: Kind of putting them in groups.
Int: And what do you do when you divide?
S3A: You take them..kind of like taking away.
Int: What's the difference between dividing and subtracting?
S3A: Umm..Dividing you take..um..you kind of do it in groups and in subtracting you do it in singles.

Similarities between addition and multiplication, and subtraction and division are described. The difference is in working with groups in multiplication and division. Not all subjects were able to explain the four operations. Only addition and subtraction were explained by younger subjects. Although they mentioned the other operations they could not explain them fully.

Int: You said you use numbers for adding up.What's adding up?
S3B: Oh..er.. division..oh doing sort of plus..er..
Int: Well how do you do it?
S3B: You sort of.. you ..er ..plus that is if you want to add 5 and 9 you count 5 and start at 9 and want 5 more from there.
Int: Right and what do you get?
S3B: 14.
Int: And how do you know that?
S3B: Ah..because we use it a lot in maths .
Int: So you know it and what about take away?
S3B: Well you have a number like 5 and you wanted to take away another number. You say, if it was 3 then you just take down 3 like you..4,3,...I mean 4, 3, 2..that's 3 down.
Int: Right well what happens to numbers when you you take them away?
S3B: They get less.
Int: What happens when you add them then?
S3B: They go bigger.

Int: What sort of things can you do with them?
S2B: Ah..you can times them, plus them, divide them, take them away and ..
Int: Well let's talk about some of those. You said you can plus them..What does that mean?
S2B: You add.
Int: Yes and what happens to numbers when you add them?
S2B: They become higher.
Int: And what about take aways?
S2B: You take away something like if you've got 12 and you take away 6, you'd have 6 left.
Int: So what happens to numbers when you take them away?
S2B: You'd have less.
Int: O.K. what about times?
S2B: You say, like, 5 times say 10 you get a group of 5 and put 10, the next group to make 10 so you're putting them in sets.

Int: Right. What happens to numbers when you times them? You put them in sets..
S2B: Ummm..
Int: You're not sure..O.K. what about divide? You also said you could divide them?
S2B: Well that's seeing how many numbers are in a number like..umm..how many twos there are in 8. There's 4.
Int: How do you you know there's 4 twos in 8?
S2B: Because half of 8 is 4.

The idea of addition resulting in a higher number and subtraction in a smaller number was appreciated by Group A and B subjects from all class groups with the exception of S1B. His explanation was more like that for Group C and D subjects.

Int: Yes and when you add things up and find the answer what are you doing?
S1B: Oh if you're having take away you find how much you're taking and if you're having 'and' you, you're putting more stuff in ('and' means add here).
Int: So if it's 'and' you put more in and what kind of number do you end up with?
S1B: Then count your numbers.. you put, you put and then you find, you do the answer.
Int: Yes and what about taking away then.
S1B: Well if you had 8 and you said take away 1 you'd have 7

Int: What else do we do with numbers then..What can we do?
F2D: We can add them.
Int: Yes and what does that mean when you add them?
F2D: It means you learn to count.. if you're young you learn to count with the numbers like if 4 plus 23, some kids might not know what it is and then they've got to count on their fingers and sometimes they take far too long.
Int: Well do you do that?
F2D: Sometimes (laughs).
Int: Well you told me you can add things up and you use your fingers when you're learning to count but what's adding .. What does it mean?
F2D: Umm..
Int: What are you doing when you add things up?
F2D: Um..putting numbers together..
Int: And what do you get?
F2D: Sometimes the right answer, sometimes the wrong answer.

Int: You said you can multiply
S4C: That's when you take like..four fours are 16 and you times them.
Int: How do you do times?
S4C: Oh drill them.
Int: Drill them do you.How do you drill them?
S4C: Like if you've done a mistake you cross it..um.. diagonally.
Int: Do you want to show me how you draw it. I'll give you some paper and then you draw it?
S4C: (Draws a multiplication sign "x").
Int: Like that, oh I see what you mean... for times.

S3D: The numbers that are used for maths..um..like in a maths book..um.. so that you can put in an answer..if you want to know some.
Int: So what can you do with the numbers at maths then?
S3D: Um..um...
Int: Well what do you do at maths time?
S3D: Learn.
Int: What..what is it that you have to do at maths time?
S3D: Umm..learn sums.
Int: What are sums?
S3D: They're..um..numbers.
Int: Yes..well tell me some sums then and that might help you to explain it.
S3D: 6..4..64 times 42.
Int: Can you do that?
S3D: Yes.
Int: So what other kinds of sums are there?
S3D: Um..um..lots of others.
Int: Well what other sorts. Tell me some.

S3D: 36 times 42.
Int: Well what does times mean?
S3D: Um..like you times that amount.
Int: Yes but what does times mean though. What are you actually doing?
S3D: Um..um..You're doubling the number that you add, you double the number.
Int: Right, what else except times, what other things can you do?
S3D: You can do divided by..um..you see how many numbers you can get in one..um..number..Like if you say um..10 divided by 2.
Int: And what's that?
S3D: 4.
Int: What else can you do then...times..divide..
S3D: Take away.
Int: O.K. tell me about take away.
S3D: Umm. Take away like if you have 3 take away 2 you're just taking away a number.
Int: Yes and what do you get?
S3D: An answer.
Int: An answer..O.K. and what else can you think of..times, divide, take away.. what else can you do?
S3D: And plus.
Int: Yes tell me about plus.
S3D: Plus is..um..adding..um..adding another number like if you had 5 plus 5 that equals 10.
Int: Oh yes and what do you get?
S3D: An answer.

The Group C and D subjects include no real explanation of the operations. These examples illustrate their mechanical use of numbers in following a known procedure to produce an answer. Although the correct answer may be obtained it has no real meaning to these subjects. Methods taught in school are followed but not understood. The next example provides further evidence of this :

Int: Well let's see. You told me...you must do some other things besides times tables at maths. What else do you do?
S2D: Hard things.
Int: Well what are hard things?
S2D: Like the, you do a straight line then a curve and you put a 6 out the side and then he puts like 54 something in the middle.
Int: Yes do you know what that is? ...Do you know what people are doing when they do that?
S2D: Err...not really.
Int: Have you done any?
S2D: Yes.
Int: Are they hard?
S2D: Yes.
Int: You don't know what they are.
S2D: But Mr. A helps everybody do it together.

This subject is describing division and gives an accurate picture of the way it is set out including an example he may have seen or worked on in the classroom but he has no idea of what it is all about. By working in a group with the teacher he may acquire the procedural knowledge needed to perform the calculation but without understanding he is unlikely to be able to use this procedure in problem solving situations. Further discussion demonstrates a similar lack of understanding.

Int: What else do you do that's easier..some things that are

really easy for you?
S2D: Family of facts.
Int: O.K.Tell me what they are.
S2D: They're like, think they're like 9 plus 3 or something like that.
Int: Yes and what's 9 plus 3?
S2D: 27 (Counts on fingers).
Int: 27 O.K. well how do you make a family of facts?
S2D: Ummm
Int: Forgotten?
S2D: Yes.
Int: Show me how you did 9 plus 3. Do it out loud so I know what you did..How did you do 9 plus 3 using your fingers like you were.
S2D: I just go 3 and 6 and then 9, then it's a bit harder. I just count like 1,2,3 (with fingers) .
Int: Oh and so you got to 27.
S2D: Yes.
Int: Now what..Have you ever done any adding up?
S2D: Yes.
Int: How do you do adding up?
S2D: Like 5, just pluses..
Int: How do you do those?
S2D: 5 plus 5
Int: And what is that?
S2D: 25.

Learning multiplication facts by repeated addition has resulted in the misconception that this procedure may be for addition as well as multiplication. Again a school taught procedure is applied without understanding the operations involved. The Standard 1 subjects have less experience of number operations than subjects from other class groups. S1D had little to offer in explaining operations.

Int: What are pluses? You said pluses what do mean by pluses?
S1D: Just adding.
Int: Adding.O.K. What do you do to numbers when you add them?
S1D: (shrugs shoulders)
Int: Do you know what that means? You add numbers up...You don't know. Well what else can you do with numbers instead of adding them up?
S1D: Count.
Int: Yes and how far can you count?
S1D: I don't do any counting.
Int: Well have a go see how far you can go.
S1D: I think I can count only up to a thousand.
Int: A thousand.
S1D: Well up to a thousand and nine or something...or nine thousand. I don't know.

Even when asked follow up questions little information is obtained and the subject appears reluctant to offer explanations. It is interesting to note the numbers offered as the limit of counting, a thousand and nine or nine thousand. Both of these are typical common errors in counting.

The main difference between those subjects with good mathematical ability and Group C and D subjects with poor mathematical ability was in their understanding of the operations. Group A and B subjects understood and applied their knowledge whereas Group B and C subjects mechanically applied school taught methods to obtain an answer. Further illustration of this lack of understanding was demonstrated in subjects' explanation of the equals sign.

F1A: Equals is when something has the same amount as something else and like you'd use it in a, in a sum like 5 plus 9..that's 14, equals 14.So 5 plus 9 is 14 and that equals 14. You can also say have it in those sums like 5x5 is equal..oh no that doesn't work.. um.. 12x2 equals 4x6 or something like that or you can do things like that and it's written by putting two little lines over the top of each other.
Int: Do you know what we call those lines?
F1A: Hyphens.
Int: No..A maths name..
F1A: Oh..parallel lines.
Int: Right and what does that mean?
F1A: It means going in the same direction for the whole time and they don't, they never meet.

Int: What's equals?
F2A: Oh well that's like when you're adding something, that's to see, this is what the problem, the answer is.
Int: O.K. So what else can we use equals for ,except what the answer is?
F2A: O.K. like say 6 plus 3 and 8 plus 1....6 plus 3 equals 8 plus 1. That's the same.
Int: The same. O.K. Can you describe equals for me..What it looks like?
F2A: Oh it's just two lines like that, (draws lines in air) two parallel lines.
Int: O.K. two parallel lines. What are they?
F2A: Oh they're lines that are equal distances apart and they go on..

1A: Hyphens.
Int: No..A maths name..
F1A: Oh..parallel lines.
Int: Right and what does that mean?
F1A: It means going in the same direction for the whole time and they don't, they never meet.

Int: What's equals?
F2A: Oh well that's like when you're adding something, that's to see, this is what the problem, the answer is.
Int: O.K. So what else can we use equals for ,except what the answer is?
F2A: O.K. like say 6 plus 3 and 8 plus 1....6 plus 3 equals 8 plus 1. That's the same.
Int: The same. O.K. Can you describe equals for me..What it looks like?
F2A: Oh it's just two lines like that, (draws lines in air) two parallel lines.
Int: O.K. two parallel lines. What are they?
F2A: Oh they're lines that are equal distances apart and they go on..

Group A subjects appreciate the equivalence of numbers but F2A initially offered "What the answer is." as an explanation. This view was very common.

Int: What's equals?
F1B: Oh ..er..it's when any number, or any two numbers um, or it could be one number ..that um.. it's when you can either plus and it's just,um..equals means what it equals, what the final result is after all your work, all your working .
Int: O.K. Anything else that equals means?
F1B Ah..um..well you can, well in a running race two people can be equal, exactly the same time over the finishing line.
Int: O.K. How can it mean that in maths...exactly the same?

F1B: Oh..er..um..Oh right..Oh..um..like for example..Oh there's greater than and smaller than..um...besides if you've got 6+4 and then 4+6 on either side then you put equals cos they equal the same thing..Otherwise if it was 6+3 and 6+4 then it would be the greater one..6+4 which is open up the mouth. (This refers to the use of the greater than sign (>) which some teachers describe as an open mouth).

Int: Do you know what equals is?
S4A: Um..well equals is just a sign that means two lines one above and one below each other and they just say that the, that the answer's about to come up or that the answer's about to come up or equals or...the answer coming up.
Int: O.K. What else do you equals for?
S4A: Oh when you say something like 28 equals 24 plus 4.
Int: What's the difference there..what's equals mean in that one?
S4A: It means that it's equal.
Int: Yes what's equal mean though?
S4A: Um..it's the same.

Int: Something else you see a lot at maths time is equals. Do you know what that is?
S2B: Umm..It means put the answer after it.
Int: O.K. Does it mean anything else?..Can you use it for anything else?
S2B: Well in..um..like..and you've got 90 equals 9 tens.
Int: And what does it mean there?
S2B: It means it's the same as.

These Group A and B subjects explained the meaning of equals as the same and gave a variety of examples to demonstrate this. This response was usually obtained after the initial interpretation of a command to put the answer next as would happen on a calculator. Other subjects from these groups had some idea of the equivalence of numbers but the command to put the answer was their common interpretation.

S4B: Umm.Equals cos it..it's like 5 and 5 equals 10 and you've got to show the equals sign to show that you've finished and..umm.. that's the answer.
Int: Well is there anything else we use equals for?
S4B: Umm...
Int: Think of another time you might need to use equals.. What does equals mean?
S4B: Like the answer to something.
Int: What does the word equal mean?
S4B: Umm like things with brackets..um. like 5 times 5..like 40 times 30 or 4 times 3 and you go..um..like 4 times 30 equals, put brackets and in the brackets you put.. um ..4 times 3 and then you put the brackets and then write plus and then in the brackets you put something else..
Int: You can't remember the last bit?
S4B: I did it in my maths book.
Int: Well what does equals mean when you do it like that?
S4B: Show the other way like you're changing it around.
Int: O.K. Can you change it to anything you want?
S4B: I don't know. You could sometimes.

S3A: Do you know what equals is?
S3A: What the two together are or whatever it is.
Int: What else do we use equals for..what does it mean.. equals?
S3A: Kind of like the answer .
Int: Yes does it mean anything else ?
S3A: They equal the same weight or the same something
Int: Well how do you use that in maths?
S3A: For weighing.
Int: Yes .How would you use it like with number sentences?
S3A: Like 2 plus 3 equals 5 .
Int: Now does that mean the same or the answer?
S3A: That's the other meaning the answer.
Int: Well can you think of the other meaning.Think about it.

S3A: 1 plus 2 equals 3 equals 3 plus 0 and they mean the same.

Int: Right and what about, you said they all equal up to something. See if you can tell me a bit more about that.
S2A: Like a ..9 plus 9 is 18 and like 2 and 2 is 4 and 3 and 3 is 6 and they all have an equal. They all equal up to something.
Int: What is equal? What does that mean?
S2A: Umm...what it equals up to.
Int: And what does that mean?
S2A: umm...umm...ummm...answer.
Int: O.K. Do you know any other meaning for that word?
S2A: Umm...Like the two numbers come together as one.

The two meanings for equals, the same and the answer is coming up, were offered as separate meanings. The equivalence of numbers in operations is not fully understood although subjects appreciate operating on numbers produces new numbers.

Int: Do you know what equals is?
S1B: It's the thing at the, it's second to last.
Int: Yes and what do we use it for?
S1B: Well, well if we had 8 and 8 equals that's what it's for..equals 16.
Int: That's where you use your equals.What does equals mean?
S1B: To put it together.

F1C: Equals is used for equations like 1 plus 1 equals 2 so equals is just saying what the numbers in front of it add up to.
Int: What does it mean?
F1C: Well equals means it equals something maybe.
Int: What does the word "equals" mean?
F1C: Equal..um..well...equal means um..equals means..Well equals just really means, well just like I said 1 plus 1 equals 2 and it just means that..um.that when you have a whole string of numbers and you add them it's got to equal something cos you can't just have 1 plus 1, 2 it wouldn't make sense.
Int: And what are equations? You said you use it in equations.
F1C: Well equations are just a word, it's a word meaning um ..that a group of numbers added, subtracted, divided by, times and it equals something.

F2C: Well it's the, it's the answer to a sum really, it's the conclusion, not the conclusion but the, really the end of it to show it's equal and not equal.
Int: What do you mean by equal and not equal?
F2C: Like say for not equals 1 plus 1 does not equal..oh..3 it equals 2 so you'd say 1 plus 1 equals 2 which means it's the total of the um objects that are there or numbers whatever and oh yes, 1 plus 4 is not equals to 3 plus 8. That's what the not equals is.
Int: And what does it mean not equals?
F2C: Well it doesn't mean it's..it doesn't add up to what the other sum says..or it's not true, it's false.
Int: It's false. O.K. and what about the equals sign?
F2C: An equals sign. Oh it's to show what it adds up to.

Int: See if you can explain what equals is.
F2D: It's something, it's something..ah..umm..it's something at the end of some numbers like 5 plus 5 equals something. It's to show you what, and sometimes it stands for um.. what it, I had it in my head before and I've forgotten now .....um...Oh, it's to show you where to put the answer..It shows you,um it tells you to add it together and put the answer down.
Int: Anything else?
F2D: Um..not really.

Int: What's equals?
S4C: It means the amount what the 2 addends or products have made.

Int: What's equals?
S4D: Well you see 5 and 5 and it's the quantity you end up with....it's what you end up with after 5 plus 5.
Int: Is there anything we use equals for?
S4D: I can't really think of any others.

Int: Do you know what equals is?
S3B: Yes.It's a ..er..er...sort of..say you had a sum and and want 8 plus 8 and then you put an equals sign and you put an answer.
Int: O.K. So what does equals mean?
S3B: Well it means you've got to ..it means the answer.

Int: You said equals before. What does equals mean ?
S3D: It's the..just tell somebody the answer.

Int: You told me equals here before..What's equals?
S1C: Equals is when you add something together and minus is when you take away and times is when you times something like 2 times 4 is 8, 2 fours are 8 just like that.
Int: Yes well why do we have equals?
S1C: Because..because that ..you put, like you set it out, you set it out kind of, I'll just do one. 3 times 3, three twos and you put equals and the teacher wants to know the answer so you put your answer like 6 is the answer.
Int: Right Is there anything else we use equals for?
S1C: Err...put equals to...you equals to..to put...
Int: Well what's it for?
S1C: Well that's the answer to it.

Int: Why do we have equals?
S1D: If I had five plus five it would have to equal something.
Int: So what's equals mean?
S1D: (shrugs).

The interpretation of equals as a command to put the answer in all these explanations demonstrates use of procedural knowledge but a lack of real understanding of the relationship between numbers.

This finding supports research described by Hughes (1986) where subjects found equations in unfamiliar form difficult to complete. For example writing 3=[], or []=1+2 are difficult for subjects who read equals as put the answer. If the equivalence of the number relationship is appreciated then there is no difficulty in understanding what is required. Instruction involving less familiar forms of equation are not sufficient to overcome this problem in understanding as shown in this final example:

Int: What do you know about equals?
F1D: Um..it's an answer ...it's like sum plus the other, add it all up and you put equals , you put the number that it adds up to.
Int: Yes.Is there anything else you that you use equals for?
F1D: Oh I can do it backwards.
Int: How would you do it backwards?
F1D: 3 equals 2 plus 1.
Int: How come you can do it backwards?
F1D: I don't know. I just remember doing it that way at my old school.....I've forgotten what it's supposed to do and you just sum backwards like 24 plus something equals..
Int: Oh yes. Anything else?
F1D: That's backwards.... um I can't think of anything else.

Experience with a different form of equation has resulted in the ability to complete this type of equation but by following a procedure, you do it backwards, not because the equivalence is now understood.

7.5 Processing Information

Many Group C and D subjects were observed using their fingers to help work out calculations during explanation of number operations.

Int: When do you use your fingers?
F2D: Um..When..if I get stuck on a number and I lose where I'm up to..if people disturb me.
Int: Yes.Do you have trouble remembering where you're up to?
F2D: Yes...sometimes I remember my 5, 10, 15, 20 and sometimes I go um 5 and then I'm meant to go 6 but I go 10 instead..I forgot what's after and then I go like that and I can remember what number's next.

Int: How else can you help, like you use your fingers, what else can you do to work things out?
F2D: Umm..use a calculator or ruler..um...that's all.

Int: Do you use anything else to help you?
F1C: Fingers.
Int: Fingers..there's nothing wrong with using fingers.. they're there..you might as well.........When do you use your fingers?
F1C: When you've got to add something quite big.
Int: You wouldn't have to use them for something like 5 plus 8?
F1C: No...13.
Int: If I said something like 46 and 9.
F1C: Oh I might get a bit confused.
Int: You might get a bit confused. What if I made it 27 and 58?
F1C: I'd probably get confused too.
Int: Well what do you do when you get confused?
F1C: I'd probably just take a little longer...to figure it out or use a scrap piece of paper..... or a calculator.
Int: Or a calculator..Do you like using a calculator?
F1C: Yes well they're quite good.

Use of fingers for these older subjects was one of the methods they suggested for difficult problems where they might also use paper or calculators as alternatives. Other subjects described how fingers were actually used.

S4C: You can use your fingers for counting.
Int: Aha..How do you do that?
S4C: Like you have them that and then you go like that.. putting them out (fist closed and then opens out one finger at a time with each count)..
Int: When do you use that?
S4C: Oh when you're stuck.
Int: When you're stuck. Can you show me when you're stuck how you do it?.. Can you think of a hard thing that you need your fingers for?..What kind of thing would you use them for..? Say I said 4 plus 5..do you need them for that?
S4C: No.
Int: O.K. so you don't need your fingers O.K. how about 8 plus 9?
S4C: Yes I will.
Int: Well you show me how you do it for 8 plus 9.
S4C: (Counts on fingers extending one finger for each count) 8,9,10,11,12,13,14,15,16,17 and that's the answer.

Int: And how do you know when to stop?
S4C: When I get up to the right amount of fingers.
Int: O.K. and how do you know when you've got there?
S4C: Well like if I want to know 7 I've still got one up (one finger and a thumb and the other hand had all 5 digits extended.)

Int: You were using your fingers there..Do you do that a lot?
S4D: Yes.
Int: Show me how you do it with your fingers.
S4D: Umm I do..oh I didn't do the bigger number first and take it from there.
Int: You take the biggest
S4D: Yes and then I went..Oh you know what's ones what that's 10,9,8,7,6,5,4,3,2,1..(shows on fingers) .
Int: Yes.
S4D: I'd just gone 17,18,19....26.
Int: Yes and that's how you do it.

Int: So if I asked you to work out say..Oh..9 plus 8..Do you know the answer?
S4D: Errr...no.
Int: How would you work that out?
S4D:........
Int: You want your fingers?..You're allowed them.
S4D: 19
Int: Not quite. How did you get 19? How did you get that,can you remember?
S4D: Not really.
Int: Well you do it on your fingers.
S4D: I probably made a mistake somewhere.
Int: Well do it on your fingers..9 plus 8..
S4D: (Counts on fingers)..10, 11, 12, 13, 14,15,16,17..17.
Int: Yes..Much better for you on your fingers isn't it. What about something like 4 plus 5?
S4D: 20.
Int: 4 plus 5.
S4D: 8
Int: How did you do that?
S4D: I didn't use my fingers.
Int: What did you do then because I saw your mouth moving.
S4D: Yes..umm..Oh usually I can do it. I go 5,6 and then on paper I usually just put 4 dots
Int: You do dots on the paper do you. I see.
S4D: Or on the side..or else I move my head. I go 5,6,7 ..(nods head for each count).
Int: So you nod your head.O.K. All these things help do they?
S4D: Umm..I usually do that.

Subjects count on from the larger addend using their fingers to keep track of the count. They begin by holding out the number of fingers required for the addend that they have to count and then indicate one count as each finger is passed over. Other methods mentioned include nodding head and writing dots on paper to keep track of counting on.

Int: Well what do you do to help yourself at maths time, working things out?
S2C: We use our rulers.
Int: Yes anything else you can use to help?
S2C: Mmmm..rods, dice..
Int: Yes what do you do mostly?
S2C: Just use my ruler and my head.
Int: How do you use your head?
S2C: Just try and remember. I just add in my head.
Int: How do you do that that?
S2C: I just think of one number that you're going to get and think of another, you can use your fingers and count on numbers.
Int: And how do you use your fingers for counting on?
S2C: Well if you've got 10 and you want to make 19 you go 10, 11, 12, 13,14,15,16,17,18,19, (touches fingers).
Int: And how do you know when to stop?

S2C: Because you come to that certain number.
Int: And how do you know when you're there?
S2C: Oh cos you count on your fingers like this. There's 1, 2,3,4,5 (touches fingers on one hand) so I know I've got to go over 5 and 5,6,7,8,9, so I've got to go over 1,2,3,4,5,6,7,8,9 so I've got to go over 5 then 4 (shows 5 fingers on one hand and then 4 on other).
Int: Let's see if you can work some out. What's 5 plus 6.
S2C: 5 plus 6.. 11.
Int: How did you do that one?
S2C: I just know that one.
Int: O.K. Well what's 5 plus 8?
S2C: 5 plus 8....wait a minute.
Int: You use your fingers.
S2C: Mmmm cos it's hard to count in your head sometimes cos you haven't really got anything to touch and you don't know how many you have to count on, like on your fingers and on your ruler you have cos you can actually see it but sometimes you can't see it in your head.

This subject again describes a counting on method with fingers but explains the need for concrete items in counting. These items, fingers or numbers on rulers can be seen and touched as the count proceeds unlike working at an abstract level when information is more easily lost.

Int: And how do you use counting to work out sums?
S1C: Er..count..umm..numbers..say 1, 2, 3, 4..
Int: How do you use that when you're working out sums?
S1C: You use your ruler because all the numbers are on your ruler...they're all up to 30.
Int: And what happens if they're past 30?
S1C: I just do it on my fingers.
Int: And how do you use your fingers?
S1C: I go like..if it's two tens..one count 10, that's 10 already (shows 10 fingers) so I count another 10..10, 11, 12,...20 (touches fingers).
Int: Right and what if it's something like 52+35?
S1C: Then I just..umm..(laughs)..do it on my fingers again.
Int: Well show me.
S1C: O.K. and so you start off with..What was it again? [52] and we end up with [and 35] .So it's a take away isn't it?.
Int: No it's adding up.
S1C: Oh yes. umm so 35 add on er.What was it? [52 add on 35] 52 so 53, 54, 55, 56, 57, 68, 59, and on my page I write down the number what I have to get to and when I get to it I stop and I get an answer.

In this example the subject has difficulty remembering the problem as she attempts to count on a large number and finally explains that she records the number to be added on paper so that it isn't forgotten.

7.6 Forgetting

Older subjects referred to problems with retaining information during calculations and this influenced the method they used.

Int: Now if you have to work out problems at maths time do you prefer to do it in your head, on paper or how? How do you like working things out?
F2A: Oh well if it's real easy well I just do it in my head otherwise I like doing it on paper.
Int: Why do you like doing it on paper?
F2A: Oh it's a lot easier, yes, you can set your work out so you can see and in your head you sort of forget things

as carrying numbers and that.

Int: I notice you've been writing on your hand (with finger) a lot, like on paper..Do you use that a lot?
F2B: If I haven't got a bit of paper in my hand.
Int: So you like the paper..why do you like writing it down so much?
F2B: Um..because you forget, you don't forget the numbers and it's easier when you can see the numbers and add it up,in your head you tend to forget what the equation is

Both these subjects prefer written methods so that information isn't lost and F2B also writes with his finger on his palm as a way of fixing data in his mind while performing mental calculations. Subjects from all groups described problems related to forgetting.

F2D: We had this test... and some of them I didn't know, cos some of them were a bit hard and I put down the wrong answer and I was going to scribble it out and I rubbed it out instead and I was going to put the number down and someone stared at me and I forgot what it was and I lost my place.
Int: Yes..It sounds like you have a bit of trouble remembering ...is it hard?
F2D: Yes.
Int: What's the problem...do you know? Do you think other people have trouble remembering?
F2D: My sister does.
Int: How about people in your class?
F2D: Umm.not that I know of..

Loss of attention and a delay in recording the appropriate answer caused problems for this subject.

Int: What do you like best at maths time?
F1C: Oh nothing.
Int: You don't like maths?
F1C: No.
Int: Why not?
F1C: I don't think I'm very good at it.
Int: You don't think you're good at it. Well what do you find hard about maths?
F1C: Well remembering all those numbers and everything..cos it's hard, cos when you do times it's hard to remember a whole set of times.
Int: You find it hard. have you always found it hard to remember?
F1C: Yes.

Again difficulties with remembering are mentioned this time learning times tables are given as an example.

Int: Why do you think we bother learning maths?
F1D: Oh otherwise you don't get ripped off. Like if you went to buy something and so, you didn't know your maths and someone knew you didn't know your maths, well they didn't have to know that..
Int: Have you been ripped off yet?
F1D: No I haven't been ripped off yet.
Int: You told me you weren't good at maths.
F1D: Yes I know but I'm not that bad at it..(laughs).. Cos I'm better at money.
Int: You're better at money.. Why are you better at money?
F1D: Cos there's nothing like, there's no one and a half pieces of dollar notes like..ah..It's there I can count it instead of making it up in my head..Making a sum in my head's quite hard.
Int: Hard to do a sum in your head..
F1D: Yes.
Int: What else can you do if it's hard to do in your head then?

F1D: I can write it down on a piece of paper.
Int: Does that help?
F1D: Yes .That helps a lot cos I can see it.
Int: O.K. Why's it hard to do it in your head? What's the problem?
F1D: Well I start to do a sum and I forget the sum.
Int: O.K. you forget the sum.
Int: How about adding up 6 plus 9 and things like that.
F1D: Ah..it's quite easy..9,10,11,12,13,14,15...(counts on six fingers) but any big number is quite hard.
Int: You like using your fingers.
F1D; Yes cos I know where I'm up to and I like to..so I can count to 9... O.K.... 1,2,3,4,5,6 and then up to 9... 10, 11, 12, 13 but I forget what number I'm up to.
Int: You have a bit of trouble remembering them.
F1D: Yes cos when I do a times table I know 6 ones is 6 but I know the answers, the beginning and the answers but the numbers 1,2,3.. I forget where I'm up to and that's where I get mucked up...where dad has to help. I know all the answers. Just the middle ones are the trouble.
Int: O.K.then if you were adding up say 5 plus 6.
F1D: Um..11
Int: How did you do that one without your fingers?
F1D: Cos I know 5 plus 5 is 10 and 1 more is 6, 1 more than 5 ..5 plus 5 is 10 and 1 more is 11.
Int: That's right. Now how about doing 6 plus 9 a quicker way than.Can you think of something that would help you work it out?
F1D: er..6 plus 9..9..um..14.
Int: No.. Never mind try this one..7 plus 6.
F1D: 13
Int: You did that one fast without your fingers.
F1D: Cos I know 6 plus 6 is 12 and 1 more is 13.
Int: Right and it's much quicker isn't it ? And your head can figure it out so you don't have to use your fingers then.
F1D: Yes cos if I know half the sum I can just plus on the other half...it's much easier.

The need for concrete counting items for keeping track is mentioned again with fingers and recording on paper suggested as ways of helping calculation. When problems close to doubles are involved, for example 6 plus 7, a different approach was used. Now prior knowledge of a fact, 6 plus 6, aids the solution resulting in a much faster completion of the problem.

7.7 Sub Vocalising

The other behaviour observed as subjects performed caculations was sub vocalising. Not all subjects did sub vocalise. Group A and B subjects generally worked silently. They were asked about thinking internally rather than expressing their thoughts aloud.

Int: Do you think in your head a lot or are you the sort of person who likes to think out loud when you're thinking?
F2A: Oh I like thinking just by myself..I can talk by myself for ages.

Int: How do you usually work things out at maths time when you're doing problems and stuff?
F1A: Quietly..What do you mean, how I work them out?..Well it depends what the problem is.

As F2A says talking in your head is common for these subjects, just something they do all the time but other subjects found this idea strange and were not aware it was possible.

Int: Do you work things out in your head?
S1C: Yes.like you get 10 bottle tops and another bottle top and you see them in your head and add another 10, so you see another 10 in your head and then you, you count them up...like and you go 1, 2, 3, 4 like that all up to 20. (Whispers number names as eyes move).
Int: So you actually see them in your mind ...like your eyes are counting them. Do you say the words?
S1C: No you go (whispers 1, 2, 3, 4) because the teacher will say "Quieten down a bit! "
Int: So you're trying to talk quietly while you're doing it but do you have to do it out loud like you're doing it now?
S1C: So you count, you go 1, 2
Int: Do you whisper like that?
S1C: Yes 1, 2, ...11, 12..
Int: So you have to keep whispering.Can't you do it without talking at all? You can imagine now that there's 7 bottle tops and there's 10 more and you try and count them and I'll watch...You're moving your lips..What happens if I say don't move your lips and see if you can still do it..
S1C: (laughs) I need a breath...
Int: Yes have a breath...You"re still moving your lips.
S1C: I can't.
Int: It's hard isn't it. Can you talk to yourself in your head without moving those lips? Say I said count to 10, count out loud to 10, off you go.
S1C: 1, 2, ...10
Int: Now you're not allowed to move those lips or move this (point to voice box) where your voice goes. O.K. now you're going to try and say that in your head without talking aloud.O.K. are you ready..go then.
S1C: (laughs) I can't.
Int: You can't ..
S1C: Yeh...
Int: You just did it. It's quite hard isn't it.Some people do that all the time. You're obviously not used to it.
S1C: No cos I use my fingers instead...it's like because a ruler isn't that much help because it only goes up to 30 and its only got the hundreds like 101,102, 103..104 and that's not much use because we don't do that cos our fingers we can count them again and again and again and again.

Sub vocalising was used continually by this subject and she had great dificulty in counting without sub vocalising. Finally after concentrating really hard and shutting out all distractions by closing her eyes and blocking her ears she reported that she had managed to do it. Using fingers with sub vocalising was used for calculations although she also described mental images of bottle tops which were counted by sub vocalising the number names as she moved her eyes as if looking at real bottle tops. She found it impossible to count without sub vocalising.

Other subjects reported they were able to think internally or out loud but when observed were seen to be sub vocalising, sometimes without being aware they were doing it.

Int: Do you use your fingers and things?
F2B: Yes when I'm adding up or..
Int: Do you find you're talking out loud without meaning to?

F2B: Yes. Work it out in my head, add something up and keep going until I get there.

Int: Do you do a lot of thinking to yourself when you work.. or do you talk out loud a lot to other people?
F2D: Yes.
Int: You prefer to talk out loud. What about when you're working things out?
F2D: Umm..sometimes I say the number or I leave it in my mind and think about it while I'm adding and stuff.
Int: Is it easy to think in your mind...so if I gave you something to work out you could do it in your head?
F2D: With a bit of help from my hands.

Int: Can you think in your head or do you tend to think out loud?
F2C: Yes I can do it out loud and I could do it in my head .
Int: Which is easiest?
F2C: Probably in my head.
Int: Do you do a lot of things in your head or do you tend to say things
F2C: I don't. If I read a book I like to read in my head not out loud.
Int: And what about when you're working things out?
F2C: Umm well when I'm working out an equation I usually do it out loud.

Reading a book is done without sub vocalising but calculations involve sub vocalising. This indicates that reading and computation involve different mental processing.Reading is comprehended in terms of existing knowledge and as long as no inconsistencies with prior knowledge built on experience occurs incoming data does not require conscious interaction with existing knowledge. In computation however if, for example, known facts are recalled and used then there is constant interaction between incoming and existing data.

This may account for differences in mental processing and explain why subjects working at a concrete level need to sub vocalise or talk out loud as they count items. They are not utilising stored knowledge in the form of known facts but counting items. Those subjects who do work at an abstract level process information internally utilising long term memory stores which gives them immediate access to data on permanent record, minimising the load on remembering external information.

The next example describes how this subject has a problem spelling words silently. Spelling may require the same mental processing as computation if known words are used to help spell unknown words. Children who have dificulty spelling may need to say the word out loud to hear the letter sounds.

Int: Do you do a lot of thinking in your head?..Or are you a person who talks aloud a lot?
F1D: When I spell something and I'm writing a story and I get a good burst and I know what I'm writing I sometimes spell out loud and I don't know that I'm spelling it out loud and it annoys my next door neighbour..
Int: Cos you don't know you're doing it. Well what about talk..Do you know how to talk to yourself without calling out aloud? Can you do it..you know if I said count to 10 you can do that can't you?
F1D: 1,2,3,4,5,6,7,8,9,10.
Int: Now can you do it without moving your mouth and your tongue...just inside your head.....
F1D: Yes.
Int: Is it easy?
F1D: Oh it's not as easy.
Int: Do you do that or talk out loud at times?
F1D: At maths I don't say it out loud at maths but when I'm writing things down I say it out loud.
Int: What do you do at maths time?...You look at the paper so you don't have to remember it in your head.
F1D: And everyone else is saying it as you write it up on the blackboard and all saying the answers..er..so I don't say it.

For subjects not used to thinking internally this was a skill that required some effort to acquire. Keeping speech muscles at rest while they counted required a lot of concentration and external distractions such as noise or movement immediately broke their concentration and they lost their place.

7.8 Visual Images

Two subjects mentioned seeing items in their head as they counted. S1C, in the extract above, described seeing an image of bottle tops and her eyes moved as she counted these imaginary items but a different image was formed by S2D.

Int: How do you work out the answers?
S2D: Umm Just work it out
Int: Well how do you do it?
S2D: Umm..Add it with your fingers or your head.
Int: How do you work things out in your head. I saw you doing it with your fingers. How do you work things out in your head?
S2D: Um..you just count in your head.
Int: Can you do that?
S2D: Yes.
Int: Do you see things when you count in your head?
S2D: Yes.
Int: What do you see?
S2D: Umm. Red stuff and green. I have my eyes shut and all green and yellow.
Int: And what do they look like those red, green and yellow things?
S2D: Just like stripes coming down.
Int: And you count those do you?
S2D: Yes.
Int: Alright. Well you close your eyes now and count some for me and I'll see you doing it...(eyes shut)..How many have you counted?
S2D: 5
Int: 5...So you have to look at the things and say the numbers in your head do you..like fingers a bit isn't it...that's a handy way..do you do that a lot?
S2D: Yes when I'm trying to get to sleep.
Int: Do you talk out loud or move your mouth?
S2D: No just think carefully.

Counting of imaginary bottle tops actually innvolved a count of a real item, eye movements. The count performed by S2D was a more abstract count and S2D did not seem to sub vocalise as he counted these items as he did when counting on his fingers. His eyes were closed as he counted as he focused his attention on the visual image. This attention to internal counting may have caused the actual count to become internalised too as he shut out the external world. He states that in this case he has to think carefully. Perhaps this is an indication of a transition between external and internal counting that may prove to be a method other children could use in switching to internal counting.

7.9 Multiplication Tables

Learning multiplication tables was mentioned by most subjects. They had all tried to learn these tables often with help from other family members. Descriptions of how subjects, who had found this a difficult activity, learned tables included the following methods.

Int: How do you learn your tables, you say you find it hard, how do you go about learning them?
F2D: Well sometimes when I've got maths homework I get my sister to test me and when I get some some of them wrong she tells me what they are and she tells me to say them about 10 times in my mind.
Int: And do you do that. Can you remember them?
F2D: Sometimes I get stuck for where I am but most times I can remember.
Int: And how do you practise saying them?
F2D: Umm..I just, if it's 10 times all of them and the answer..I might say a hundred and it's wrong..um.. and she tells me the answer and I go like that and say.. (moves lips).
Int: So you're saying it in your mind? Your mouth's moving isn't it?
F2D: Is it?
Int: So you're saying it with your lips too.
F2D: It's hard for some people and some people find it easy. Some people, it takes them a long time.
Int: Why do you think that is?
F2D: Cos they're not quite sure what the answer might be and they work it out in their heads sort of for numbers.

Reciting isolated facts over and over as described by F2D was one method of learning tables. Sub vocalising is used in repeating the fact and the subject was unaware of doing this. Other subjects also spoke of taking a long time to learn tables.

Int: So it's always been hard. Do you know your tables now?
F1C: Oh ..most of them.
Int: Which ones do you know the best?
F1C: Um..ones, twos, fives, sevens, nines, elevens, tens.
Int: Which ones are hard?
F1C: Um..Fours can be sometimes, sixes are O.K....not the best ..um..nines can be tricky sometimes and um.. threes are alright.
Int: Well how do you try to learn them? What do you actually do when you're trying to learn them?
F1C: Well what I usually did is I repeated them in my mind or if I got stuck on one I'd write it on my hand and every so often when I remembered about it I'd try to remember the number and well I'd look at my hand to see if it was right.
Int: And how do you do it in your mind?
F1C: Well....I just sort of just keep on saying it to myself and just keep on saying it.
Int: Just in your mind.
F1C: No.
Int: You move your mouth or just use your mind?
F1C: I just use my mind.
Int: Do you find you're always working quietly or are you talking as you work them out?
F1C: Well I sort of use my mouth (laughs)..
Int: You do..That doesn't mean the words come out though does it?
F1C: It just sort of as you say it to yourself and your lips are moving at the same time.

Again working on isolated facts is reported this time writing it on the hand so the correct fact can be checked throughout the day until it is known. This subject describes sub vocalising as she works on learning facts. S4D also sub vocalises and finds it difficult to stop doing this.

Int: Well what's hard about learning your tables?
S4D: Just getting yourself to sit there and do it.
Int: Can you think of a quicker way instead of having to remember them?
S4D: Oh remembering them is the quickest way you can get.
Int: That's the only way you can think of is it?
S4D: Ah..apart from using your calculator and um... sometimes you get some bottle tops and use them and that's about all.
Int: Well do you use things like bottle tops at school.
S4D: Yes we use them at school.
Int: Bottle tops and stuff like that..anything else?
S4D: Umm...I've got a tables chart..it goes from the twelves to the zeros....I can't think of anymore.
Int: Do you think in your head a lot or do you like to think out loud?
S4D: Oh..er..no. I like to think in between.
Int: How do you mean in between?
S4D: Sort of like softly.
Int: Do you move your lips but not talk..is that what you mean?
S4D: Sort of.
Int: You kind of say it softly. Is that what you mean?
S4D: Yes.
Int: O.K. Now can you count out loud to 10..just start and count 1....10. Now when you talk you can feel your voice here. Now what I want you to do without moving your lips, and without moving that is to count to 10 in your head. O.K. you see if you can do it...and you have to actually say the numbers without saying them out loud ... Can you do it ?. Keep your lips together and try... oh mm..you're still talking out loud.
S4D: I can't do it.
Int: You can't do it ..never mind. So you had to talk out loud..You just couldn't do it without talking out loud. Do you want another try? You have to concentrate. Some people shut their eyes and try and shut out all the noise and they gum up their mouth and try and do it..
S4D: (shuts eyes and tightly closes mouth) .Yes.....ah..Can you do it?
Int: Yes I can do it.

To eliminate sub vocalising all other distractions have been reduced by closing eyes and blocking ears yet this subject still has to really concentrate on counting in his head. The approach taken to learning facts is again to memorise them. The next example describes problems with memorising facts.

Int: What sort of things have you been doing for your homework?
S2C: Just the basic tables
Int: And how good are you on those?
S2C: Not very good
Int: Aren't you. Why's that?
S2C: Because they don't stick.
Int: You get stuck.
S2C: No. They don't stick.
Int: Because they don't stick.
S2C: I can't remember them.
Int: Why not?
S2C: I don't know.
Int: You don't know,it just happens. Which ones do you know?
S2C: My two times, my five times, and my 10 times.
Int: And what are you trying to learn now?
S2C: Ah..my 8 times.
Int: And how do you do it when you try and learn.What do you do to try and help yourself learn?
S2C: I keep on going over and over and over and over and over.
Int: Do you do that.
S2C: What you do...about 1 each night you know and do it and keep on saying it to yourself until you've got it.
Int: Show me how..say it.

S2C: Like when you read you go (whispering) 5 times 10 is 50, 5 times 10 is 50 and you keep on going like that.

Learning isolated facts as these subjects have done is a time consuming experience. Other subjects described a different method where they learned groups of facts and used known facts to work out unknown facts until they eventually had developed good recall of all facts in the multiplication tables.

7.10 Prior Knowledge

Using known facts involves prior knowledge that is stored in long term memory. The following examples show how known facts are used in working out unknown facts.

S4B: Like if I couldn't get it, 5 times 5 which is 25, I'd say 5 times 4 and add on 5..I'd say 25.
Int: Do you do that a lot?
S4B: I don't know.
Int: Do you know most of the tables? Were they hard to learn?
S4B: They were hard at first but they're easy now.
Int: And how did you learn them?
S4B: Um..At first what I did is I would just add on and add again and again and I'd know how to do it and I'd practise them every night.
Int: What did you do when you were practising them?
S4B: I was thinking of what I'd done before and..um..you see it was like.. I'd work them out but I already knew them at school and that and also I'd just remember the answers when I was doing them and then I knew how to do them.
Int: Do you think out loud or in your head?
S4B: Think in my head.
Int: Do you think a lot? Are you a person who thinks a lot or do you tend to talk when you're working things out?
S4B: I don't talk.

Int: O.K. now you said something about times tables.
S3B: Well times tables you...it's like ..just like plus..it's like 6 times..er..you add it up normally 6 and 6 and 6..18... Then if you get one like seven sevens..sort of you can't get it off by heart..by adding..6 and 6 and 6..er..um..well...you say 6 times tables..say doing..you're doing 6 up to 7 times and then you add 7 and that gives the answer.
Int: Can you work it out. What 7 times 6 is or seven sevens?
S3B: It's 49.
Int: How do you know?
S3B: I just know that's all

Well known multiplication facts are recalled and then adjusted by adding on the appropriate number so if six sevens is known then adding on a seven will give seven sevens. Prior knowledge was also used in addition and subtraction example as shown by S3B.

Int: How do you work things out?
S3B: Oh well I use..er.my fingers.
Int: Yes .Do you do that a lot?
S3B: Er..no..
Int: Just when do you use them?
S3B: Well say..er..I was stuck on something..you know you've gone wrong somewhere and you've got to try and figure it out..
Int: Yes. What else do you do . When you don't use your fingers?

S3B: You could just use your brain.
Int: Do you do that a lot? How do you use your brain?
S3B: Well I just .er..see say there was one that had equaled 20 something and I couldn't sort of get it..well..I would sort of add in just small numbers like say I couldn't..couldn't get it so I'd go on....say if it was...say..kind of count in twos, tens or something else like that.

Subject knowledge of numbers and renaming numbers is used when adding on a large number by performing the addition in parts. This procedure reduces the problem to one that the subject can then manage using counting in twos or tens.

It is those subjects, mainly from Groups C and D, who attempt to learn facts in isolation and sub vocalise. Other subjects learn groups of facts, working on one times table or using patterns in the numbers. Subjects who work at an abstract level have access to prior knowledge stored in memory and can use this to work out new facts until they too become stored in memory. Data is organised in memory because the relationship between facts has been part of the process of learning these facts. Grouping facts reduces the units or chunks of data to remember and also provides access to whole groups of facts when recall from memory is required.

7.11 Problem Solving

The ultimate aim of learning number operations is that they are to be used in problem solving situations. Subjects who successfully completed number problems were given verbal problems to solve to observe strategies they used. Mental arithmetic was used to solve most problems and this involved methods illustrated by the following examples.

Int: Now when you have a problem to work out like say, there were 42 children and the bus would take 80, how many empty seats would there be? How would you work it out?
F2C: Oh,umm, I'd just concentrate..38.
Int: How did you work that out?
F2C: Oh, um, you have to add on how many it would take to get from 42 onwards up to, how many did you say? [80] How many up to 80, see how many there were on your working out paper.
Int: Well how would you do that on a piece of paper or in your head like you just did it?
F2C: You could, yes, cos it's quite easy to work it out cos it's just 10, 10, 8.....10, 10, 10, 8. It's just um, like 42, you plus it by 8 and then you 30,30 more to make 80..so you'd say 38.
Int: Well if you can do that in your head why do we learn to write it all down on paper?
F2C: So we can learn to learn it in our heads.
Int: Does it help you to learn it in your head by writing it on paper?
F2C: Yes you keep writing it down sort of for practice and then it turns up in your head.
Int: O.K. then if I write it down on this piece of paper, will you show me how to do write it down if you're

working it out on this piece of paper?
F2C: Umm.... and then..I don't know how to..um..if there's 80 people on the bus..I don't know...
Int: This was real easy in your head wasn't it?
F2C: (Laughs) Yes. .um..42 and then I'd just see how many numbers would add up to....I'd probably just work it out in my head. I wouldn't write it down on paper.. umm..I'd go....
Int: Do you use your head a lot at maths time?
F2C: If it's a simple one like 4 or 8 or something I'd go like that quickly or um, if it's not I'd write it down like I'd go 10+10...8..I'd go 8+10+10+10 and add that up and that'd be 38.
Int: That'd be 38. That's right...How did you learn to do it that way? Who taught you to do it that...?
F2C: Oh just the textbook.
Int: The text book.
F2C: Yes..um..I just see it written down and um..an example and do it that way. So I don't think I'd work it out that way.
Int: Cos I can't remember seeing a text book writing it down that way.
F2C: Neither can I (laughs) I don't do it that way I just sort of figure it out in my head. It's just easy.
Int: It is isn't it.
F2C: Unless it's like 87 and 113.
Int: How would you do that one?
F2C: Umm I'd count up to it like, unless it was 513 and 42. I'd count how many hundreds in between and count..
Int: See if you can do it. Write down the two numbers.
F2C: What was the other 513?
Int: Yes.
F2C: I'd figure out how many digits to get to the hundred and then 60, no, 48 no it's not that's 90..(laughs) 42...yes that's 90, um, 42..58 right and that's 100. So 58, that's 58, so you reach 500 and I go 200, 300, 400, 500 so that's 558. So 558 plus 13 which is 500...558....and that's..no that's 458 so I'd just sort of do it that way and 13 is..
Int: It's getting really complicated isn't it?
F2C: Yes 71. So it would be 471.
Int: What's that for? (Points to 1 next to tens digit written when adding 13 to 458.)
F2C: If I put any of them down here, so you'd put down the 1 in the ones compartment and 11 here in the tens compartment up here and when you added 5+1 that would be 6 you'd add this one here as a 1 and it'd be 7 .
Int: So just that'd be 7 would it?
F2C: Yes.
Int: That one's worth 7 is it?
F2C: Oh 70. So it would be 471. Yes 471.
Int: Right.
F2C: That's the difference.
Int: Right write it down then, 471 if you think that's the answer. And can you think of a way of writing it down, a subtraction way..
F2C: Oh yes..513-42. That would give you the difference.
Int: Try it and see if it does.
F2C: (Uses working form) 471 Oh I forgot. I would have done it that way actually.
Int: You forgot all about it didn't you. When you see problems that are in words, as I gave it to you, the children and the bus with empty seats, do you always try and work it out in your head?
F2C: Yes I usually try to. If it gets too difficult, um, like that one I just did I'd write it down like that.
Int: But you didn't think of writing it down like that did you? Do you find it hard doing word problems...are they hard or easy?
F2C: Well when you write it down I can think about it a bit more. I can look at the question over and over, say, I find problems quite easy sometimes unless they get quite difficult like, like how many fences go round 18 in perimeter or whatever.
Int: What makes them hard...D'you know?
F2C: Umm...just the wording I think. They try and um, make your brain sort of off the line of the sum, off the track and they turn you away from it. If it was simple, unless they're quite hard sometimes, they're written so I ask the teacher and she'll explain it and I'll try and work it out from there. That's how I do them..

This subject was able to use school taught written methods with all types of addition and subtraction and yet did not consider using this method of calculation when solving the problem. Instead a counting on procedure was used and although long and quite complex in keeping track of all the steps involved the correct solution was finally produced. This subject persisted with her own informal method even when prompted to write it down and also referred to this method being seen in a text book but later changed this saying it was easy to work out things in her head.

The first problem about the bus was easily solved by an informal counting method but writing down the solution proved very difficult. The school taught written method was not used. Most subjects used the counting on method for the problem with the bus and empty seats but other older subjects changed to written methods with the more difficult problem once keeping track of data became difficult. A further example of this preference for informal methods was shown by S4A.

Informal methods were used in problem solving rather than the school taught written method by subjects from all class groups. Only Group A and B subjects attempted the more difficult problem as the other subjects found number problems of this complexity difficult to complete. Subjects could describe and successfully complete number problems using school taught methods but failed to apply them in problem solving.

Int: How do you work things out when you've got a problem to solve?
S4A: Well I do the 5 step method, figure out what the problem is in the first place.then you figure out what numbers or sets you use and the.. you work out what the sum's going to be, then you work out the answer and then you figure out some words to answer like there'll be 5 apples remaining or whatever.
Int: O.K. Alright then. Who taught you that?
S4A: Oh I need to do it at maths sometimes but we usually do it in the shorter form ,just the three step method.. there's the answer and then the writing answer or something...there's 6 apples left or whatever.
Int: O.K.if I give you some bigger numbers ..like there's a stereo for sale and it costs $501 and you've saved up $287 .How much more money do you need?
S4A: Er..287..er..$314.
Int: And how did you work that out?
S4A: First I did..287 was it? I added it up 87 plus 14 was it? 14 plus 87 equals 301..then plus another 200 more ... I keep forgetting what the number was.
Int: Cos I slowed you down........
Int: Well you didn't do that by any 5 step or 3 step method did you?
S4A: Not really no. I usually do if there's one, if there's a problem like..when there's a real hard method I usually do it by the 3 step method but like that I usually do the 2 or 1 step method.

7.12 Attitudes and Reading Difficulties

Group A and B subjects generally enjoyed maths and experienced few problems with this subject at school.

Int: How do you like maths?
F2B: Umm..sometimes I like it, other times I don't It can get a bit boring...but it's fun sometimes. I like doing shapes and things like that, drawing things. I don't like percentages and when something's hard to explain and you know the answer but you can't explain what it is and when the questions are real confusing and I can't..It's hard to get the answer and work it out um..
Int: Why are percentages so hard?
F2B: I don't know.I can get, usually I can pick it up and remember it but I just....I know how to do it and then I forget again .

Success in a subject would generate enjoyment but finding concepts hard to understand resulting in confusion and difficulty in remembering procedures as described here would be frustrating. It is easy to understand how those subjects who continually have trouble remembering procedures and such things as times tables would not rate this subject high on their list of favourite subjects. A change in school can also have an effect on attitudes.

Int: Do you like maths?
F2C: Um..yes, I suppose I do. It's not my best subject,it's quite fun to learn things as long as it doesn't get too difficult..like binary numbers.
Int: Those are hard?
F2C: Yes.
Int: Did you find it hard changing over schools?
F2C: Yes...to do with maths...yes cos we never really did maths. We only did about one subject of it a week and now we do it every day and I find that quite difficult. We do everything much harder too but now I've got used to it.
Int: Did you find there were things you hadn't done before that you should have done...that you didn't know?
F2C: Yes.
Int: Can you remember what kind of things it was that you had trouble with?
F2C: Oh even last year in Form 1 I didn't know my times tables very well because they didn't teach it. They thought you'd just know it, um, er integers and decimal currency and things like that. We should have known that but.. never taught us. They just thought you knew it.
Int: So they gave you some help or did you have to work it out on your own? Did you get some help?
F2C: In Timaru?
Int: Here.
F2C: Oh yes I did.
Int: And now you think you've caught up quite quickly.
F2C: Yes. I'm glad I've shifted up here.

Learning times tables appears to have received some emphasis at this school. Several subjects talked about it as a task they had to work on over quite a long time in some cases and other family members have been involved in helping to learn tables. This was not the case at the school in Timaru. Also more topics appear to be covered in the mathematics programme at the

Christchurch school. F2C clearly had difficulties adjusting at first but has been helped to catch up in those areas. Subjects who experienced real difficulty with mathewmatics did not enjoy this subject as S1D states.

S1D: I hate doing counting and I hate doing maths as well... except for today.
Int: What did you do today then?
S1D: Oh just this stuff, I don't know what it was about. She didn't even tell us what to do.
Int: But you managed to do it.
SID: A wee bit.
Int: Yes..What did you have to do?
S1D: It was a wee bit hard.
Int: Yes ..What was it then?
S1D: Ah..just a big number sentence and some pluses and take aways and everything.
Int: And you had to work them out. Well what are pluses and take aways?
S1D: Three of them each and there was just five plus three and stuff like that and we had to put the pluses together and make them equals and then put a wee comma and then carry on with the other two and then we went back and started to do them again with the take aways.
Int: Oh yes and you managed to do some of them.
S1D: Ah..I only got up to number two...

This subject's lack of understanding and attempts to follow procedures taught are obvious from her comments.

One subject became interested in the study I was doing and spoke about her observations of other pupils in mathematics lessons.

F1A: Maths is something a lot of people don't do very well.
Int: It's hard to help the people who aren't very good.
F1A: Yes cos it just sort of clicks for people.
Int: Do you know why..why..that's what I'm trying to find out. Do you know why some people are so good and other people just can't see it?
F1A: Cos they can't can they.
Int: To you it's obvious
F1A: No they don't understand it at all.
Int: Some of them can read quite well.
F1A: It's just the subject that some people click to and some don't.
Int: Yes .People think in different ways.
F1A: Some people in my class, I mean they don't understand anything and some people can..it's very obvious ... Some people find , people find it really hard to learn their times tables and things and some people..
Int: Learn it almost straight away and you learned yours quickly.

The comment about some people finding it obvious while others don't understand it at all summarises the differences between those subjects with good and poor mathematical ability. Group A and B subjects understand and apply their knowledge while Group C and D subjects mechanically perform procedures.

One group of particular interest was those subjects who had good language/poor mathematics ability. Subjects were also asked about their interest and ability in reading and other language subjects.

Int: What do you like best at school?
F2A: Um..probably writing, reading and writing.

S1A: I like printing, cos you learn to print neater.
S1A: Just stories cos you can get to write any adventures and stuff and one more..one more..I had one more. What was it? There's so much things we do at school.
Int: What about reading?
S1A: Yes I do like that, only half because sometimes I do get it right but sometimes not all the time and I like art.
Int: What do you mean, at reading you sometimes get it right and sometimes..
S1A: Because some questions are hard when you do S.R.A. If there were questions like..Bill and Jill and it would be br--- or dr--- something to the beach..
Int: Oh yes and you've got to guess the word...But do you like reading books?
S1A: Yes.
Int: And you're good at reading books..
S1A: Like we've got heaps and heaps at home.
Int: And what about spelling?
S1A: Yes I do like that....but I've got to get some words off the teacher.

Int: And what's your favourite subject?
F2C: Spelling.
Int: Are you a good speller?
F2C: Yes. I like spelling a lot.
Int: Do you do a lot of reading?
F2C: Yes, yes quite a lot, about .. in a couple of weeks.. I try to any way..Yes I read every night, I read a lot.. it's good.

Subjects in Groups A and C had good language ability. the three examples given are typical of their responses. They enjoy reading and read a lot. Writing and spelling were also popular with many of these subjects. Some Group B subjects enjoyed some reading material but not books.

Int: What do you like best at school?
F2B: Umm..I like P.E and I like, not reading books but reading projects and things like that.
Int: Do you read a lot?
F2B: No I don't read much but I like doing projects and things and um, science is alright..um..social studies..

Int: What do you like best out of anything you do at school?
S4B: Does it have to be in the classroom?
Int: No. Anything.
S4B: Netball and reading.
Int: You're a good reader. What's your favourite sort of book?
S4B: Romances,

Reading material enjoyed by these subjects was sshort in volume of text and of high personal interest. They did not read widely or frequently as Group A and C subjects did. Other Group B and D subjects did not find any reading a pleasurable activity.

Int: You really like maths, what else?
F2D: Well I don't like reading.
Int: And why do you hate reading?
F2D: Oh cos it makes me eyes get all tired when they read cos..keep on going like that (rubs eyes) and my eyes move from side to side and it hurts my eyes.

Int: What about when you see word problems in books? What are they like?
S4D: Well they stump me a bit.
Int: Do they?
S4D: Yes. I'm used to the number problems and all that.

Int: Why do they stump you if they put them in words?
S4D: Umm..Oh..I don't like reading much.
Int: What's your problem with reading?
S4D: I don't really know.
Int: Why's reading hard for you? I presume it's hard if you don't like it?
S4D: Yes..yes..Oh I don't like reading much. It's O.K. if I'm reading a wildlife book but I can't get stuck right into a..book like that.. it's O.K. if it's I'm reading a comic strip.
Int: You can read that?
S4D: Yes but like a Dr Who book, I'm O.K. on that but I couldn't get right into it.
Int: Why can't you get into books?
S4D: Oh I don't know. it's probably because I don't like reading much.
Int: Is it because you can't read the words, does it get tiring?
S4D: Yes, it's tiring on my eyes.
Int: Do you have any trouble keeping your place?
S4D: Yes I do a bit when I go from line..sometimes I read one line and then I go back to the other line and I read the same line again.

Int: How do you keep your place then when you read?
S4D: Oh....
Int: Do you keep your finger on or anything?
S4D: No not really.

These subjects describe problems that occur when they read making their eyes tired as they lose their place while reading. Research into eye movements has investigated this problem of tracking across and down the page. Weak muscles around the eye result in a loss of control over eye movement as the eyes travel across the page and down to the next line. This makes reading tiring as the muscles are weak. One of these two subjects has now been referred to a specialist to correct this problem. Other Group B and D subjects may have similar problems.

Int: Do you read much?
S3B: Not a lot but I read some.
Int: Why don't you read a lot?
S3B: Well you see I don't want to get too bored of it. It's one of my worst subjects.
Int: Why's reading one of your worst subjects?
S3B: Er..So you see when I can't work out a word I sort of get used to it and then I try to fit in some other word and it's not it and then I forget all about it and have to start all over again.
Int: What about math..do you forget things at maths time too?
S3B: Not always.
Int: Not always. It took you a long time to learn your tables though didn't it. Do you loose your place when you read?
S3B: Yes.
Int: And what do you do when you loose your place?
S3B: Oh I start using my finger....I think..I'm really ..see .. when I can't work it out I get all frustrated and I sort of can't work it out.
Int: So reading's your worst subject. Are you getting some help?
S3B: Yes my mum's helping me on reading.

S2B: I'm good at spelling and writing but I'm not that good with my reading.
Int: Why not? What's the problem?
S2B; I don't read enough.
Int: Well there must be a reason why you don't read enough. You obviously don't like it if you read enough.
S2B: Well I like playing and doing other things.....
Int: What's it like when you try to read books..What makes

it hard for you?
S2B: I don't know.
Int: You don't know. It's just hard. How do you keep your place on the page?
S2B: I just get a piece of paper and just.. cut out a bookmark shape and use that.
Int: What happens if you don't use a piece of paper?
S2B: Umm. You could buy a bookmark.
Int: What happens if you don't use anything when you read?
S2B: You just try and remember the page number.

Int: Do you have any trouble looking at the words or keeping your place when you're following the words?
S2D: I just keep my finger on or use a piece of paper.
Int: Do you, how do you use a piece of paper?
S2D: My mum puts the piece of paper along the line and then she just moves it down
Int: Yes and does that help?
S2D: Yes.
Int: What happens if you don't do that?
S2D: I just get all muddled up and angry.
Int: Do you..why?
S2D: Because mum just, say we're trying to figure out the word and she says it and I get mad.
Int: Before you've had time.
S2D: Yes.

Int: Why do you read out loud instead of reading them out quietly?
S1D: Cos, cos, If I don't read it quietly..cos if I read it quietly in my head then I just miss lines and can't read it.
Int: And does reading it out loud help you. So you find you're reading things out loud .Do you just move your mouth or do you say the words?
S1D: I sometimes say the words, sometimes move my lips.
Int: So you're a good reader.
S1D: Yes even if I do miss lines and things.
Int: How come you miss lines?
S1D: I don't know.
Int: And do the numbers get jumbled up at maths time too.
S1D: Yeh.

These subjects also reported losing their place as they read. Fingers were used to keep track of where they were on the page. Using a piece of paper and moving it down a line at a time was another way of helping subjects to keep their place. All found reading a frustrating experience.

Simple eye movement tests (Duffy and Kernahan 1987) were conducted on the subjects who had reported problems with keeping their place as they read. This involved them following the point of a pencil with their eyes as it was moved across their face, around in circles and a figure of eight. These procedures are described in "Daily Perceptual Motor Training Activities' (Duffy and Kernahan,1987). Almost all subjects tested had some problem following the pencil and a more thorough check may be worthwhile to investigate possible problems that may be corrected with specialist treatment.

This eye tracking problem can also cause problems in reading numerals in mathematics as digits are read in the wrong order thus changing a problem which may be interpreted as the result of a careless mistake by the teacher.

The analysis of data from the tests and individual interviews suggested some differences between subjects with good and poor mathematical ability.

The series of tests administered showed prior knowledge, pattern recognition and addition/subtraction of ten to numbers were more closely related to subjects' ability to complete number problems than was performance on memory and counting sequence tests.

The further series of tests which investigated the effect of the time interval between members of an item suggested that subjects of below average mathematical ability made no use of the time interval on those tests where information had to be recalled in a different form to that presented. More able subjects appeared to use this short time to process the data in some way.

More detailed information related to how subjects processed information was obtained during the interviews. Group A and B subjects, with good mathematical ability, demonstrated an understanding of number and number operations which they could apply in problem solving using mental or written methods.

These subjects used prior knowledge when recalling number facts, learning facts, estimating, and performing operations demonstrating an abstract level of thinking. They were able to apply their understanding of the number system when adding or subtracting ten by using the place value structure of numbers and able to recognise and then continue number patterns because they understood the relationships between numbers. Their own equations written in Test I showed they could apply rules and generalisations rather than remember lists of facts. Over time they have built up a well organised knowledge of numbers that is easily recalled and applied to a variety of situations.

Group C and D subjects, with poor mathematical ability showed no understanding of number and number operations. They used procedural knowledge to complete number operations which they found difficult to apply in problem solving. Informal methods based on counting procedures were used rather than the school taught procedures that they were able to mechanically follow. Counting was the base of all work with numbers for this group of subjects and featured prominently in their discussion.

Fingers frequently aided calculations and subjects sub vocalised as they counted.

The poor mathematical ability subjects appeared to be operating at a concrete level, needing items to count and using counting procedures to solve problems. Common errors made by these subjects on the add/subtract ten and pattern recognition tests showed they were using their counting knowledge, writing the next two counting numbers after the last number in the pattern given to them and making errors consistent with common stopping points in the standard counting sequence, for example increasing the hundreds digit rather than the tens digit when zero was in the tens place. Sub vocalising was common among these groups of subjects and together with their emphasis on counting suggests that they do not think internally but externally. As a result they do not have ready access to prior knowledge which has to accessed from long term memory.

This raises the question of how information is accessed. Group A and B subjects reported talking to themselves "in their heads" and frequently engaged in this activity regarding it as quite natural. Group C and D subjects did not find this to be the case. If subjects are able to carry on a silent conversation with themselves then it is possible that they access stored data by asking questions such as "What do I know already? How should I do it? What can I think of that would help me? " Thinking becomes a concious activity and allows interaction between stored and incoming data.

A closer examination of the interview transcripts showed that Group A and B subjects often made noises like "umm" when pausing before continuing with their ideas. This was less frequent for Group C and D subjects and usually followed a question or prompt to continue from the interviewer. In her study of Children's Learning Alton-Lee (1984) observed this subject behaviour and suggested it indicated thinking was occurring. In these interviews the two different ability groups' behaviour suggests that the able subjects instigate their own internal thinking while the interviewer prompts the other subjects to think further. This supports the idea that long term memory may be accessed by self questionning.

Reading appears to require a different kind of processing. Unlike computation and problem solving concious interaction between stored and incoming information occurs only when conflict between the two exists and then the reasder begins to question what they have read and if that appears correct their previous ideas. Again questions are asked in resolving the conflict.

Group B and D subjects, with poor language ability, generally were found to have problems following the print making reading a tiring and frustrating experience .

In conclusion the underlying difference betwen subjects in the four ability groups was a tracking problem for the poor language subjects and processing information internally or externally for subjects with good or poor mathematical ability. A remedial programme was developed on the basis of these findings.

CHAPTER 8

REMEDIAL PROGRAMME DEVELOPMENT AND IMPLEMENTATION

The main underlying difference between subjects with good and poor mathematical ability was the ability of those with good mathematical skills in abstract terms. A programme (T) was designed to help children develop abstract thinking skills so that subjects with poor mathematical ability could use these skills to improve their understanding and application of number and number operations in problem solving situations.

8.1 Programme Aims

1. To develop abstract thinking in subjects with poor mathematical ability.
2. To help those subjects apply abstract thinking skills in performing number operations and problem solving.

8.2 Programme Development

Implementing the findings of research into classroom programmes often presents some problems. Teachers may be reluctant to change, programmes may be adapted to meet teacher needs and classroom factors such as management and organisation of groups may make implementation difficult. Although the ideal situation may be to withdraw individual children for a period of time, as happens with reading recovery programmes, this involves extra funding for schools and those children withdrawn miss out on some classroom activities. In addition only a small number of children may take part in these type of programmes and special teacher training is required.

As a teacher the researcher aimed to develop a programme that would be attractive to teachers through out primary schools and easy to implement in ordinary classrooms with a minimum of teacher training and additional resources. To meet these requirements a number of factors had to be considered:

1. Which mathematical abilities to include.
2. How to identify subjects with poor mathematical ability.
3. How to diagnose common and individual problems.
4. How to plan activities to meet individual subjects' needs.

5. How to ensure the programme was appropriate for a range of subject ages.
6. How to make the prograsmme enjoyable for subjects and teachers.
7. How to achieve positive learning outcomes.
8. How to measure of gains made.
9. How to conduct trials in ordinary classroom setting.
10. Provision of activities for other children needing extension.
11. Teacher training and participation in the programme.
12. How to maintain effective communication with teachers .
13. How to ensure there was minimum interference with normal programmes.
14. Use of resources available in the school.
15. How to make maximum use ot the limited time available.

These factors are discussed in the planning of the programme and in the design of test materials.

8.3 Programme Planning

There are too many different concepts in the mathematics syllabus to cover in a short trial programme. The main study tests and interview data had focused on children's understanding of number and information processing appeared to be the important underlying factor for distinguishing between subjects with different mathematical ability. If more effective information processing is developed this should be applicable across all mathematical concepts as it involves a change in thinking processes rather than acquisition of new mathematical concepts.

Remedial instruction was to focus around learning of the basic addition and subtraction facts and their application to problem solving. Considerable recent research has been conducted in this area and there is general agreement on the development of counting skills and strategies used in addition and subtraction. As subjects progress from 'counting all' to 'counting on' and use known facts in addition and subtraction operations their information processing procedures become more abstract.

Subjects of similar mathematical ability demonstrated similar information processing irrespective of their age, that is older subjects with poor mathematical ability were still operating at a concrete level while younger subjects with good mathematical ability were operating at an abstract level. The focus chosen would provide the means to develop a remedial programme applicable to subjects of all ages as addition and subtraction are performed across all class levels.

In order to assess the effectiveness of the trial remedial programme (T) an alternative programme (M) also had to be developed for use with a control group of subjects. It was decided to base this programme on methods of processing information currently taught in primary schools.

Subjects in the main study had described how they learned multiplication tables, mainly through memorisation of tables and individual facts, and these methods could be adapted to learning addition and subtraction facts. The 3 or 5 step method of problem solving had also been mentioned in the interviews. This involves following a number of steps in sorting out data and answering the problem and is consistent with the mechanical application of procedural knowledge usually carried out by subjects with poor mathematical ability. This problem solving method would form the second part of Programme M in contrast to the self questionning/ use of prior knowledge approach taken in the trial programme.

Having selected a focus for the two programmes a number of different activities were planned to cater for three main parts of the programme. Instruction of new procedures or discussion of concepts was important if new skills were to be developed or misconceptions clarified. This involved whole class or group work with the teacher and often used concrete materials.

Practice and maintenance of skills being developed were achieved in a variety of ways including games, puzzles, written exercises, writing problems and working with a partner on an oral activity. (Details of games and puzzles are given in Appendix E). Evaluation of progress made involved teacher observation, self testing, testing in pairs, short class tests and the post test.

The organisation of activities within each programme was done on a weekly basis to meet subject needs. An extension programme was included for Group III subjects so that they could work mainly on independent activities releasing the teacher to work with the remedial group.

Activities for subjects in Programme T were chosen to develop skills of self interrogation so that prior knowledge could be accessed and used. They included using known facts, related facts as in family of facts, and number patterns. Activities for Programme M were chosen to help children leaarn facts by memorisation and practice.

8.4 Pre and Post Test Materials

The main study showed that subjects who work at an abstract level recall facts or use known facts to solve unknown facts but less able subjects rely on counting procedures to solve unknown facts. This second method is less efficient taking longer to perform and being more likely to produce errors due to counting errors. A timed test involving completion of addition and subtraction facts would indicate differences in subject ability and allow common and individual subject errors to be identified.

The test was constructed using 54 addition and 54 subtraction facts as shown in Table 13. To make the test short enough for younger subjects to have sufficient opportunity to complete most of the facts yet still keep older subjects working at a fast pace subjects who completed all the facts were then to find the sum for each of the six addition columns and six subtraction columns and record each total at the bottom of each column.

Table 13

Addition and Subtraction Facts Test Items

	1+1=	2+3=	1+6=	4+4=	0+2=	1+2=
	2+7=	0+1=	2+8=	0+3=	5+5=	3+4=
	0+9=	3+3=	1+4=	2+5=	1+3=	0+5=
	3+5=	2+6=	0+7=	1+8=	2+4=	4+6=
	0+6=	1+5=	3+6=	2+2=	1+7=	3+7=
	4+5=	6+6=	2+9=	5+6=	7+7=	0+4=
	3+9=	3+8=	0+8=	4+8=	5+9=	6+8=
	8+8=	5+7=	4+9=	9+9=	4+7=	0+10=
	7+9=___	8+9=___	6+7=___	6+8=___	5+8=___	7+8=___
Total	___	___	___	___	___	___
	1-1=	3-0=	4-2=	5-4=	2-0=	4-4=
	3-2=	6-3=	2-1=	1-0=	5-3=	10-5=
	5-0=	2-2=	7-5=	3-3=	6-4=	9-8=
	6-5=	7-0=	8-4=	9-3=	5-5=	4-3=
	10-6=	8-7=	9-9=	7-4=	8-0=	9-7=
	11-8=	10-8=	8-6=	10-9=	12-6=	18-9=
	10-7=	16-8=	11-6=	12-7=	14-8=	11-4=
	12-8=	13-5=	15-8=	13-4=	16-7=	14-5=
	14-7=___	15-9=___	12-3=___	11-9=___	13-7=___	17-9=___
Total	___	___	___	___	___	___

A group of five adults, with good arithmetical skills, completed the basic facts test in just under five minutes. They were able to recall all facts without need to work them out and found totals for the columns using mental calculation. A time limit of 5 minutes was set for the test as a result of the small pilot test with adults who worked at an abstract level.

Subjects with poor mathematical ability mechanically applied procedural knowledge to problem solving whereas subjects with good mathematical ability could apply their knowledge and understanding of number and number operations when solving problems. To assess this use of procedural or prior knowledge based on understanding ten verbal questions were constructed.

Research has shown that children use informal methods based on counting procedures when solving problems. A test including verbal problems was required to take account of this method of problem solving but this presented difficulties because a great many items would be needed if all the factors influencing difficulty of verbal problems were to assessed. A short test was required in this programme and the difficulty of items as a result of their language was not of main interest here. What was needed was a set of questions that verbally described the small set of number equations used in the main study. These included finding a sum, missing addend, unknown initial quantity and the equivalence of two operations.

<u>Word Problems</u>

1. Four friends went fishing and caught 3 big fish and 5 small fish.
   How many fish did they catch altogether?

2. Two children made 10 toffee apples and gave 7 of them to their friends.
   How many toffee apples were left?

3. The teacher put 9 boys and 7 girls into four teams for a game.
   How many children played the game?

4. Nine children went to a party and had a treasure hunt. 14 prizes were hidden and they found only 8.
   How many prizes were not found?

5. When the teacher looked at Jonathon's maths book he had finished 7 problems. Ten minutes later he had finished 16 problems.
   How many problems did he complete in those ten minutes?

6. In a rugby game against Australia, New Zealand scored 6 points in the second half and won the game by 15 points to 39.
   How many points did New Zealand score in the first half of the game?

7. The children in three classrooms had lost 13 library books. After they had searched their desks they found some of the books but 5 were still missing.
   How many books were found?

8. A teacher brought a bag of tennis balls to school for children to use at playtime. By three o'clock 7 balls had been lost so only 9 balls were still in the bag.
   How many balls had the teacher brought to school?

9. The hare and the tortoise were having a race. After 9 km the hare stopped for a ten minute rest. When the tortoise had gone 7km she knew there were still 5km left to go.
How far from the finish line was the hare when he stopped for a rest?

10. Six children had a game of marbles. At the end of the game Karen and Hemi had the same number of marbles. Hemi had lost 5 marbles in the game but Karen, who started with 6 marbles had won 3.
How many marbles did Hemi have at the beginning of the game?

The ten questions constructed matched as closely as possible the structure of the five levels of addition and subtraction items from Tests J and K in the main study. Questions 1 and 2 were matched to Level 1 items in Tests J and K, questions 3 and 4 to Level 2 items, questions 5 and 6 to Level 3 items, questions 7 and 8 to Level 4 items and questions 9 and 10 to Level 5 items. The content of each question was based on experiences children would be familiar with and the vocabulary used was keep as simple and concise as possible. To identify subjects who operated on numbers irrespective of the actual problem one irrelevant piece of information was given in each question.

These two tests were used as a pre and post test to assess gains made and to identify common and individual subject errors. The pre test data was used to group children for instructional purposes.

The basic facts test and verbal problems were trialled on the subjects from the pilot study group. No problems occurred in test administration and the time limit of five minutes appeared to be suitable for obtaining a range of scores that could be used for grouping subjects in the trial programme at the school that had participated in the main study testing.

8.5 Subjects

All children in ten classes from the school that took part in the main study testing participated in the trial programme. The number of subjects varied between 260 and 280 due to absences and children leaving or being admitted to the school. There were a core of 262 children who took both pre and post tests in the sample of main interest in evaluating the programme. This included subjects from Standard 1 to Form 2. The ten classroom teachers were divided into two groups of five so that some subjects from each class level were in both groups. One teacher group used the trial programme (T) and the other teacher group used the contol programme (M).

Subjects were allocated to one of three instructional groups on the basis of their pre-test results. As older subjects tended to score more correct answers than younger Standard 1 and 2 subjects different cut off points had to be used to establish groups. A good score on the word problem test was taken as better than 7/10 for Standard 3 to Form 2 subjects, but better than 5/10 for Standard 1 and 2 subjects. A good score on the addition and subtraction facts test was taken as greater than 100/108 for older subjects but greater than 54/108 for the younger Standard 1 and 2 subjects.

In this way subjects who made common errors or failed to complete the mean number of facts for their class level were assigned to the remedial group for learning facts and subjects who failed on the higher level word problems were assigned to the remedial group for instruction on problem solving.

The distribution of subjects in each programme and class instructional group is shown in Table14.

Table 14

Distribution of Subjects within Teaching Groups

Class Group	Class Level	Teaching Programme	Teaching Group I	II	III	Total Subjects
1	Std. 1	M	2	6	5	13
2	Std. 1	T	3	13	3	19
3	Std. 1/2	M	8	12	11	31
4	Std. 2	T	3	12	17	32
5	Std. 3	T	3	15	16	34
6	Std. 3/4	M	5	10	16	31
7	Std. 4	T	2	12	19	33
8	Form 1	M	3	5	20	28
9	Form 1/2	T	3	10	13	26
10	Form 2	M	5	9	17	31

Teaching Programme T-Trial Programme
Teaching Programme M-Control Programme
Teaching Group I - Poor on Problems, Good on Facts
Teaching Group II - Poor on Facts
Teaching Group III - Good on Facts and Problems

Subjects who obtained a low score on the word problem test but a good score on the facts test were allocated to Group I, those with a low score on the facts test to Group II, and those with a good score on both tests to Group III. The subjects with a good score on the word problems but low score on the facts test were in Group II as interviews in the main study had shown these subjects used informal problem solving methods based on counting procedures. Their information processing method appeared more like,to be more like Group II than Group III subjects.

8.6 Programme Outline

The programme was trialled in the third term of 1989 and the main features of the programme included:

1. Introductory visit to explain programme to teachers.
2. Pre-test involving addition/subtraction facts and word problems.
3. Marking and analysis of test results.
4. Identification of common errors.
5. Assign subjects to ability groups and trial (T) or control programme(M).
6. Run programme over six weeks:-
   Set a weeks work based on needs of children.
   Teachers implement programme.
   Researcher marks work and monitors progress.
   Evaluate work completed at the end of the week.
   Plan the next week's programme on the basis of evaluation of previous week's work.
7. Post-test at the end of six weeks instruction.
8. Data analysis and programme evaluation.

Early in the term the researcher attended a staff meeting to discuss some findings of the main study testing and to explain the programme they were about to begin. The trial and control programme were offered as alternative programmes that should both assist children's learning. Teachers were given the opportunity to ask questions but did not know which programme they would work on at this time. The first week's instructions told teachers they were following Programme T or M. A daily time allocation of 5-10 minutes was to be arranged to suit individual class programmes.

As the school organisation included composite classes an interchange programme was in operation for mathematics lessons. After discussion with the teachers it was decided to run the programmes with their own class rather than their maths groups. This allowed children from each class level to take part in the trial and control programme and shared ability groups among all teachers.

The programme began with the pre-test in the third week of the term and concluded with the post-test in mid November. The results of these tests were given to the class teachers for their records. All materials and instructions were provided by the researcher but the class teachers administered tests and the instructional programmes. To enable ongoing progress to be monitored by the researcher subjects were each given a booklet for written work and this was collected and marked at the end of every week.

The researcher delivered the booklets and instructions for the week on a Monday morning and collected booklets on a Friday after school. One teacher was responsible for collecting and distributing material within the school. Teachers could discuss the programme with the researcher on a Friday after school before the next week's programme was planned. In this way teachers could be involved in planning the programme and making any changes they felt necessary. After the post-test teachers were asked to complete an evaluation of the programme (See Appendix F for example).

Each week's instructions were written in detail for daily lessons. An example of the detailed instructions for the introductory activity in the trial and control programme are given in Appendix D. A summary of the two programmes follows.

8.7 Summary of Programme T

Aim: To develop abstract thinking skills.
To apply these skills in problem solving.

Week 1:
Objectives:

To develop internal thinking by:
Talking silently to yourself,
Asking questions and answering them yourself,

Activities:

1.1. Subjects count silently in their head.
1.2 Practise asking and answering questions like, "Do I know anything that will help me?"
1.3. Practise use of known facts to work out unknown facts. (Begin with doubles and add/subtract 1)
1.4. Evaluate progress by means of an oral test of addition facts that may be solved using knowledge of doubles or add/subtract 1.

Week 2:

Objectives:

To maintain skills introduced in Week 1.
To develop understanding of addition/subtraction.
To determine the effect of add/subtract zero.

Activities:

2.1 Practise counting in head.
2.2 Practise asking and answering own questions.
2.3 Use objects to show addition/subtraction and how they are reverse operations. Record as family of facts.
2.4 Use objects to show the effect of add/subtract zero.
2.5 Complete family of facts examples, particularly with zero.

2.6 Play 'Basic Facts Housie' game. Use facts close to known facts.
2.7 Use alphabet code activity for independent work.
2.8 Evaluate progress -10 by 10 addition grid. Complete as much of the grid as possible in 5 minutes. Early finishers sum totals for rows and columns.

Week 3:

Objectives:

To maintain skills introduced in Weeks 1 and 2.
To develop understanding of the written code of arithmetic.

Activities:

3.1 Practise counting in head.
3.2 Practise asking and answering own questions.
3.3 Introduce written symbols used in addition/subtraction by using objects hidden under containers. Demonstrate the need for labels so quantities under each container are remembered then use addition/subtraction sign and numeral to show an operation that has subsequently been carried out but not seen by subjects who should still be able to tell how many objects are now under each container and describe in words what has occurred.
3.4 Record observed operations performed using objects in written code.
3.5 Send a message to a partner telling him/her what actions to perform to match a written number sentence that only the instructor sees. Show the written code when the operation is complete and check it was correctly performed.
3.6 Write 'What am I?' puzzles. Get someone else to solve it.
3.7 Play 'Basic facts Housie'.
3.8 Practise subtraction facts as a written independent activity.
3.9 Evaluate progress- 10 by 10 addition grid in 5 minutes.

Week 4*:

Objectives:

To maintain skills introduced in Weeks 1,2 and 3.
To investigate the purpose and meaning of 'equals'.
To develop problem solving skills.

Activities:

4.1 Practise counting in head.
4.2 Practise asking and answering own questions.
4.3 Discuss 'equals' as meaning 'the same' not 'put the answer'. Demonstrate with groups of objects, as on a balance.
4.4 Analyse a verbal problem by asking questions like, 'What information am I told, what else do I know that could be helpful, what is here at the beginning of the story-what happened- what was there at the end?"
Write down the operation in number sentence form.
4.5 Make up a simple story in words to describe a given number sentence.
4.6 Practise subtraction facts as a written independent activity.
4.7 Play 'Basic Facts Noughts and Crosses' with subtraction facts.
4.8 Evaluate Progress- 10 by 10 addition grid in 5 minutes.

Week 5:*

Objectives:

To maintain skills introduced in Weeks 1, 2, 3 and 4.
To solve problems where a difference is involved.

Activities:

5.1 Practise counting in head.
5.2 Practise asking and answering own questions.
5.3 Discuss informal methods based on counting and school written methods of problem solving. Investigate advantages/disadvantages of each method. Show how a difference can be found by counting up or back and how this relates to adding up or subtracting from the larger quantity.
5.4 Write word stories to match missing addend number sentences.
5.5 Play 'Beat the Clock' and 'Complete the Wheel' games.

5.6 Complete missing addend written examples as an independent activity.
5.7 Evaluate progress- 10 by 10 addition grid, early finishers try a subtraction grid. Sums are written at the head of each column and rows must be subtracted from columns so no negative numbers are obtained.

Week 6*:

Objectives:

To maintain skills introduced in Weeks 1, 2, 3, 4 and 5.
To solve more complex verbal problems.

Activities:

6.1 Practise counting in head.
6.2 Practise asking and answering own questions.
6.3 Discuss ways of helping subjects to 'see what is happening' in a problem by drawing a sketch, using objects, set diagrams or a picture in their mind.
Try to write the equation that describes the problem using 'x' as the unknown. Ask what was there, what happened , what is there now or what am comparing...?
6.4 Practise solving word problems.
6.5 Write your own problems for a friend to solve.
6.6 Evaluate progress-10 by 10 addition and subtraction grids. Early finishers sum column totals.

*Week 4: Standard 1 and 2 subjects continued work on zero as several subjects were still finding this difficult. Problem solving was introduced in Week 5 and finding a difference in Week 6 for these subjects. More complex problem solving was not included in their programme.

8.8 Summary of Programme M

(Details of games and puzzles are given in Appendix E.)

Aim: To improve recall of addition/subtraction facts.
To develop problem soving skills using 3 or 5 step method.

Week 1:

Objectives:

To discuss ways of helping to learn facts.
To identify personal areas of weakness.
To investigate the effect of add/subtract zero.
To practise addition and subtraction facts.

Activities:

1.1 Discuss ways of learning facts-recite, write, tests..
1.2 Make personal lists of facts to learn.
1.3 Practise addition/subtraction facts on own.
1.4 Demonstrate effect of add/subtraction of zero.
1.5 Write family of facts examples, particularly those with zero.
1.6 Demonstrate how addition and subtraction are reverse operations and how this relates to family of facts.
1.7 Play 'Basic Facts Noughts and Crosses'.
1.8 Oral test on basic facts.

Week 2:

Objectives:

To maintain skills introduced in Week 1.
To describe different types of number- odd, even, fractions...

Activities:

2.1 To practise basic facts on own.
2.2 To work on learning facts from personal lists.
2.3 To order different types of number on a number line.
2.4 Play 'Basic Facts Housie'.
2.5 Alphabet code activity.
2.6 Evaluate progress-10 by 10 addition grid in 5 minutes. Early finishers sum totals for rows and columns.

Week 3:

Objectives:

To maintain skills introduced in Weeks 1 and 2.
To solve number problems involving a difference.

Activities:

3.1 To practise basic facts on own.
3.2 To work on learning facts from personal lists.
3.3 To order different types of number on a number line.
3.4 Use > and < in number sentences.
3.5 Practise subtraction facts - independent written activity.
3.6 Write 'What am I?' puzzles for a friend to solve.
3.7 Play 'Basic Facts Housie'.
3.8 Play 'Basic Facts Noughts and Crosses'.
3.9 Use working form for two column addition/subtraction.
3.10 Evaluate progress- 10 by 10 addition grid in 5 minutes,+totals.

Week 4:

Objectives:

To maintain skills introduced in Weeks 1, 2 and 3
To develop problem solving skills using 3 or 5 step method.

Activities:

4.1 To practise basic facts on own.
4.2 To work on learning facts from personal lists.
4.3 Introduce 3 and 5 step method-identify sets involved, select appropriate operation, perform operation, obtain solution and answer question in words.
4.4 Practise word problems.
4.5 Write 'What am I?' puzzles.
4.6 Play 'Beat the Clock' game.
4.7 Play 'Basic Facts Housie'.
4.8 Evaluate progress- 10 by 10 addition grid in 5 minutes plus totals.

Week 5:

Objectives:

To maintain skills introduced in Weeks 1, 2, 3 and 4.
To solve problems involving a difference.

Activities:

5.1 To practise basic facts on own.
5.2 To work on learning facts from personal lists.
5.3 To demonstrate use of 3 or 5 step method for solving a problem where a diference is involved.
5.4 Practise solving problems with a difference.
5.5 Play 'Beat the Clock' game.
5.6 Play 'Basic Facts Housie'.
5.7 Evaluate progress- 10 by 10 addition grid in 5 minutes, early finishers try a subtraction grid. Sums are written at the head of each column and rows must be subtracted from columns so no negative numbers are obtained.

Week 6:

Objectives:

To maintain skills introduced in Weeks 1, 2, 3, 4 and 5.
To solve more complex word problems.

Activities:

6.1 To practise basic facts on own.
6.2 To work on learning facts from personal lists.
6.3 To demonstrate use of 3 or 5 step method for solving a problem where more than one operation is involved .
6.4 Practise solving problems.
6.5 Play 'Beat the Clock' game.
6.6 Play 'Basic Facts Housie'.
6.7 Evaluate progress- 10 by 10 addition grid in 5 minutes plus totals.

Those subjects in Group III who had scored well on facts and word problems in the pre-test worked on an extension programme while the teachers worked with the other groups. This extension programme involved more complex problem solving and work with fractions and percentages for Standard 3 and 4, Form 1 and 2 subjects but two column addition and subtraction for younger subjects. Subjects in Programme T or M extension groups still were involved in whole class discussions where the method of working was discussed. Programme T subjects were still asked to develop use of prior knowledge and self questionning techniques which were to be applied to solving complex verbal problems while Programme M subjects were encouraged to increase their speed of recall of facts and to use the 3 or 5 step method for problem solving.

Subjects in Group I who performed well on facts but poorly on word problems in the pre-test worked with Group II subjects for problem solving but with Group III for work on basic facts.

EVALUATION OF PROGRAMME

For analysis of the test data three groupings were considered: the whole sample, a low group and a high group of subjects. The pre-test results showed a number of subjects who scored at a high level on both the basic facts and word problem tests and so they could not make large gains as a result of the programme. These subjects in Group III had participated in extension activities during the programme.

The low group consisted of subjects from Group I and II in the programme and were able to make gains. The distribution of subjects who took both pre and post test are shown in Table 15.

Table 15

Distribution of Subjects in Pre-Test and Post-Test

Group	Class level	High and Low Groups Male	Female	Total	High Group Male	Female	Total	Low Group Male	Female	Total
1	Std.1	5	4	9	0	0	0	5	4	9
2	Std.1	14	4	18	0	0	0	14	4	18
3	Std.1	2	15	17	0	0	0	2	15	17
	Std.2	7	6	13	1	0	1	6	6	12
4	Std.2	14	18	32	0	2	2	14	16	30
5	Std.3	19	11	30	8	4	12	11	7	18
6	Std.3	7	7	14	2	4	6	5	3	8
	Std.4	7	8	15	7	2	9	0	6	6
7	Std.4	16	16	32	10	6	16	6	10	16
8	Form 1	10	18	28	6	13	19	4	5	9
9	Form 1	6	4	10	2	3	5	4	1	5
	Form 2	6	8	14	3	4	7	3	4	7
10	Form 2	11	19	30	10	11	21	1	8	9
Total Subjects		124	138	262	49	49	98	75	89	164

As the trial and control programmes were conducted by class teachers the numbers of subjects in each group varied according to differences in class size and composition. Some classes had uneven numbers of male and female subjects and data was examined to determine any overall difference for male and female subjects but no consistent difference was found. Subjects in composite classes were assigned to instructional groups on the results of the pre-test and not class level.

The mean scores obtained during pre-test and post-test for the whole group of subjects at each class level in trial (T) and control (M) programmes are summarised in Table 16.

Table 16

Summary of Test Mean Scores for Programme M and Programme T

Class Group	Prog. Group	Class Level	Problems Pre-test	Problems Post-test	Problems Gain	Facts Pre-test	Facts Post-test	Facts Gain	Column totals Pre-test	Column totals Post-test	Column totals Gain
1	M	Std.1	4.3	4.7	+0.4	43/50	55/62	+12/12	-	-	-
2	T	Std.1	4.5	5.5	+1.0	46/48	43/48	-3/0	-	-	-
3	M	Std.1	4.5	6.0	+1.5	53/57	65/71	+12/14	-	-	-
3	M	Std.2	5.9	7.0	+1.1	62/64	74/77	+12/13	-	-	-
4	T	Std.2	6.3	7.7	+1.4	59/65	100/103	+41/38	-	-	-
5	T	Std.3	7.1	8.8	+1.7	91/95	93/95	+2/0	-	0.5/0.5	0.5/0.5
6	M	Std.3	6.5	7.2	+0.7	91/94	98/101	+7/7	-	-	-
6	M	Std.4	7.9	8.7	+0.8	99/100	101/103	+2/3	-	-	-
7	T	Std.4	7.8	8.7	+0.9	91/99	104/105	+13/6	-	1/1	1/1
8	M	Form 1	8.5	9.4	+0.9	101/102	105/106	+4/4	1/2	3/4	2/2
9	T	Form 1	8.3	8.9	+0.6	90/92	100/102	+10/10	1.5/1.5	2/3	0.5/1.5
9	T	Form 2	8.7	9.0	+0.3	96/98	108/108	+12/10	0.5/1.0	3/4	2.5/3
10	M	Form 2	8.3	9.1	+0.8	106/108	107/108	+1/0	3/6	4/6	1/0

Mean scores for items correct out of total number of items attempted are given for each class level on the basic facts test. For example a score of 43/50 for Class Group 1 means this group of subjects scored a mean of 43 items correct and attempted a mean of 50 out of a possible 108 items. Recording scores in this way includes information about the number of items attempted as well as those correctly answered. As this was a timed test this information is needed in interpreting rate of completing items.

There were twelve columns to sum for subjects who completed all the fact items. A mean score of 4/6 for Class Group 10 indicates that these subjects correctly totalled a mean of 4 out 6 columns attempted. Mean scores for the word problem test are given as a score out of ten.

All but one class group showed a gain in mean scores from pre-test to post-test. Class group 2 showed a small loss on the basic facts test but discussion with the teacher revealed that test conditions were not ideal in the post-test which was administered on a hot day at a busy time of the term. Two boys became distressed during the test and the time allowed for answering questions was short as the test had to fit in between other activities. The results for this group are therefore not reliable and the Standard 1 groups are not included in further discussion of post-test results.

In the pre-test mean scores on the word problem test are similar for the two groups at each class level. Standard 1 subjects answered a mean of about 4.5 questions correctly, Standard 2 a mean of about 6, Standard 3 a mean of about 7, Standard 4 a mean of about 8 and Form 1 and 2 subjects a mean of about 8.5 correct. In the post-test all subjects from Standard 4 to Form 2 Class Group 5 correctly answered a mean of about 9 questions correctly and inspection of answer sheets showed that the last question posed difficulty for many subjects. This was the most difficult question involving an unknown initial quantity and required two equations to be balanced to find the solution. Younger subjects and Class Group 6, Standard 3 subjects showed a gain on this test but did not achieve the high success rate of the other groups.

The basic facts test shows a high completion and success rate for subjects above Standard 2. Class group 4 showed a large gain from pre-test to post-test. The pre-test score of 59/65 indicates that subjects were making several errors as they answered items but on the post-test many more items are attempted and also correctly answered (100/103).

Common errors made by all subjects formed the basis of planning. These included problems with zero , for example recording 2+0=0 or 3-0=0 , adding subtraction facts and multiplying addition facts like 2+5=10. Other errors noted appeared to be the result of counting errors as the answer given was one away from the correct answer, for example 6+8=13.

Class Group 4 which made the largest gain on the basic facts test had a large group of subjects (8/32) who made these kind of common errors, particularly with zero. Much time was devoted to clearing these misconceptions in their programme and it achieved the desired result. The other Standard 2 group in Class Group 3 did similar activities but did not show the same gain in mean score.They were in the control programme M.

Subjects in the trial programme T showed a small better overall gain than their paired subjects in the control programme M but a ceiling effect was evident for the older subjects. Differences are not likely to be significant so the data was rexamined and subjects divided into two groups. The results for high group subjects (Group III in the instructional programme) are summarised in Table 17.

Table 17

Summary of Test Results for the High Group Subjects

Class Group	Prog. Group	Class Level	Problems Pre-test	Problems Post-test	Gain	Facts Pre-test	Facts Post-test	Gain	Column totals Pre-test	Column totals Post-test	Gain
3	M	Std.2	10.0	9.0	-1.0	107/108	108/108	1/0	-	-	-
4	T	Std.2	8.0	8.0	0	108/108	108/108	0/0	-	1/1	1/1
5	T	Std.3	8.2	8.7	+0.5	107/108	108/108	1/0	0.5/1.5	1/1.5	0.5/0
6	M	Std.3	8.7	8.7	0	106/108	106/108	0/0	-	-	-
6	M	Std.4	9.4	9.6	+0.2	107/108	108/108	1/0	-	-	-
7	T	Std.4	8.7	9.1	+0.4	105/106	105/107	0/1	0/0	1/2	1/2
8	M	Form 1	9.1	9.6	+0.5	107/108	108/108	1/0	2/3	4.5/5	2.5/2
9	T	Form 1	8.6	8.8	+0.2	103/104	107/108	4/4	3/3	4/5	1/2
9	T	Form 2	9.4	9.0	-0.4	107/108	107/108	0/0	1/1	6/6	5/5
10	M	Form 2	9.1	9.3	+0.2	106/108	107/108	1/0	4/7	5/7	1/0

This group of subjects scored well on both tests in the pre-test and differences between pre-test and post-test are small. Subjects in the trial programme T attempted more column totals in the post-test and appeared to have increased their speed during the programme. The two senior class groups, 8 and 10, in programme M showed a small increase but not as great as the gain made by Class Group 9.

The low group, Groups I and II in the instructional programme, made larger gains as shown in Table 18.

Table 18

Summary of Mean Test Scores for Low Group Subjects

Class Group	Prog. Group	Class Level	Problems Pre-test	Problems Post-test	Gain	Facts Pre-test	Facts Post-test	Gain	Column totals Pre-test	Column totals Post-test	Gain
3	M	Std.2	5.5	6.2	+0.7	58/60	71/75	+13/15	-	-	-
4	T	Std.2	6.2	7.7	+1.5	56/62	99/102	+43/40	-	-	-
5	T	Std.3	6.3	8.3	+2.0	80/84	84/85	+4/1	-	-	-
6	M	Std.3	4.9	6.1	+1.2	79/84	92/95	+13/11	-	-	-
6	M	Std.4	5.8	7.5	+1.7	86/87	95/98	+9/11	-	-	-
7	T	Std.4	7.0	8.2	+1.2	91/92	103/104	+12/12	0/0	0.5/1	0.5/1
8	M	Form 1	7.2	8.9	+1.7	88/91	100/102	+12/11	0/0	1/2	1/2
9	T	Form 1	8.0	9.0	+1.0	77/77	95/96	+18/19	-	0/0	0/0
9	T	Form 2	8.0	8.9	+0.9	86/87	108/108	+22/21	-	1/1	1/1
10	M	Form 2	6.2	8.8	+2.6	105/108	108/108	+3/0	2.5/4	2.5/3	0/-1

A ceiling effect again exists for older subjects on the post-test as Form 1 and 2 subjects all answer a mean of about 9 word problems correctly. Form 2 subjects scored the total possible on the 108 fac but Programme T subjects made a gain while the scores for programme subjects are very similar on both facts and calculating totals for t columns. Overall programme T subjects showed slightly greater gai than Programme M subjects but differences are not likely to significant. Class Group 5 subjects only showed a small gain in t number of facts completed but they have fewer errors in the post tes Examination of the test paper showed most subjects had made gai between the two tests but seven subjects had worked at a slower b more accurate rate and this lowered the overall mean score. O subject scored 77/105 on the pre-test and 57/62 on the post-test a this had an impact on the mean scores.

There is variation among gains made by each class group and t difference between paired groups is not great in some cases. T difficulty of individual facts and word problems was not the sam With practice both groups were expected to show an improvement at t end of the programme but the trial programme subjects should be mo successful on the difficult items as they begin to think at abstract level.

The test data was re-examined in terms of attaining a maste level, that is the percentage of subjects who score at a high level both tests, above 7 on the word problem test and 100/108 or better the basic facts test. The results of this analysis are given in Tab 19.

Table 19

Percentage of High Scoring Subjects on the Two Tests

Class Group	Prog. Group	Class Level	Problems Pre-test	Problems Post-test	Problems Gain	Facts Pre-test	Facts Post-test	Facts Gain	Column totals Pre-test	Column totals Post-test	Column totals Gain
3	M	Std.2	15	39	+24	8	15	+7	0	0	0
4	T	Std.2	34	53	+19	6	72	+66	0	3	+3
5	T	Std.3	53	93	+40	43	50	+7	7	25	+18
6	M	Std.3	43	57	+14	64	71	+7	7	0	-7
	M	Std.4	67	87	+20	73	73	0	0	0	0
7	T	Std.4	63	91	+28	59	88	+29	3	38	+33
8	M	Form 1	82	100	+18	79	86	+7	36	68	+32
9	T	Form 1	80	100	+20	40	80	+40	20	50	+30
	T	Form 2	79	100	+21	57	100	+43	21	64	+43
10	M	Form2	73	100	+27	93	100	+7	80	87	+7

A ceiling effect again occurred for the word problem test wit all Form 1 and 2 subjects performing at a high level in the post test

More Programme T subjects than Programme M subjects scored at a high level on the word problems with Class Group 5 showing a large gain, an increase of 40% scoring at the high level but other group gains were lower, between 14 and 28% more subjects attaining mastery levels.

Similarly more subjects from the trial programme T reached the mastery level on the basic facts test but there was a ceiling effect for Form 2 subjects. The percentage of subjects in each programme who totalled columns increased more in the post-test for trial than control programme. Only Form 1 subjects in Class Group 8 showed an increase in the post-test and this class group included many Group III subjects. When gains in both facts completed and columns totalled are considered as a measure of progress made in working faster and completing more examples the trial programme subjects appear to have made better progress.

The results obtained from the tests suggested that subjects in the trial Programme T made progress working faster than subjects in Programme M although both groups improved. This improvement was supported by on going monitoring of subjects' booklets. Common errors were being corrected and grids completed each week showed faster times and more answers completed by younger subjects.

Teacher evaluations described learning outcomes for all subjects particularly in correcting common errors. Subjects in Programme M continued to use counting procedures but teachers in the trial programme noted a change in behaviour as subjects stopped counting on their fingers and used known facts. They felt a change to thinking at an abstract level was occurring. This effect was more noticeable with older subjects. Although the more able subjects in Class Group 2 enjoyed and benefitted from the programme this Standard 1 group included a large number of less able subjects who were slow to grasp what was required. Once the younger subjects understood what was required they coped well with the programme.

Grouping of subjects on the basis of pre-test results was generally appropriate for the instructional groups with only minor changes being made by three teachers. Some teachers preferred to work with the whole class and did not always use the groups. The short test was useful in identifying common errors and grouping subjects. Teacher comment included the need for grouping so that different needs can be met.

The main teacher criticism of the programme was the time involved, longer than the 5 to 10 minutes was needed for some days' activities and teachers would have preferred to include these activities in their mathematics programmes rather than making a special time in their programme. Time was needed to read through the instructions and sometimes understand exactly what was required. This was difficult on Mondays in particular when the instruction sheets were handed out on the first day of the week's programme but this couldnot be avoided as planning had to be done in the weekends.

Communication between researcher and teachers both in written instructions and availability for discussion was considered good and the activities provided were popular with teachers and subjects. These activities will be used in future programmes at the school and three teachers commented on successful changes in subjects' thinking. This emphasis on mental processing was felt to be something lacking in the textbooks and traditional programmes taught in schools. One teacher commented that one benefit of the trial programme was the move away from the procedures involved in practising the four rules to children having to think.

In general both programmes were successful in terms of providing a simple diagnostic test for classroom administration, raising teacher awareness of common errors in addition and subtraction, introducing a variety of activities for future classroom use and stressing the need for problem solving skills.

The trial programme was beneficial for both teachers and subjects raising awareness of thinking skills and the need to move away from following steps in a procedure to applying prior knowledge to problem solving. In a short time subjects began to stop counting on fingers and use their prior knowledge. The teachers felt able to adapt the programme so that they could use it within their own mathematics programmes. Although gains measured were comparable with most control groups there was some indication that given more time progress would continue if the programme was followed over a longer time period.

CHAPTER 10

IMPLICATIONS

The main difference between subjects who participated in this study was the type of information processing used . Good mathematical ability subjects used abstract procedures based on application of prior knowledge while poor mathematical ability subjects continued to work at a concrete level using counting procedures in calculation and problem solving. Older less able subjects in Form 2 still used finger counting while younger Standard 1 able subjects were using recall and knowledge of known facts in calculation.

10.1 Cognitive Development and Schema Theory

Able subjects were developing a schema for number and they recognised and used relationships between numbers. The less able subjects had gaps in their schema of number and had not yet begun to link their limited ideas about number so that they could see the relationships between numbers. Their procedure orientated number schema resulted in their understanding of solving number problems being to find an answer, hopefully a correct one. Use of procedural knowledge hindered poor mathematical ability subjects' skill development in estimation because they sought a correct answer by carrying out known steps in a familiar method. This has implications for calculator work in particular as many children will press buttons to find an answer without knowing that the answer obtained is valid or not.

These findings are important as they add to an understanding of how memory systems work. The proposed model described in Chapter 2 was supported by the results of this study. Of particular significance are the ideas of organising memory in terms of the basic universal units of time, space and matter together with reasoning and procedural knowledge, a time limit on memory capacity, and the major controlling role given to the self schema.

There are still many unanswered questions about the development of schema but the ideas discussed here offer a new way of looking at the organisation of information in general terms that could be applied to many different contexts and so opens up a whole field of topics for future study. The role of in exploring memory systems because it includes attitudes and interests and background factors such as access to resouces and expectations that influence personal learning.

Observations of subjects during interviews supported the hypothesis of good mathematical ability subjects being able to use internal speech and self interrogation in abstract information processing while poor ability subjects had some difficulty in thinking silently. These skills are used in all types of problem solving not only in mathematics and the important interaction between prior knowledge and incoming data is crucial in interpreting and solving the problem. Attempts to improve problem solving by teaching problem solving skills have often failed in the long term because skills gained have not transferred to other situations that are slightly different from the context of examples in the teaching programme.

Prior knowledge of the new context must be developed as well as problem solving skills but more importantly subjects who follow procedures regardless of context are not accessing prior knowledge as part of problem interpretation. The concious interaction of prior knowledge and incoming data appears to be a significant factor in successful problem solving and further investigation is needed to explore this interaction.

10.2 Number Concept Development

Counting played a major role in many subjects' schema of number and less able subjects used counting procedures in problem solving. Counting skills appear to be important but there has to be a move to counting of abstract units as described by Steffe, von Glasersfeld, Richards and Cobb (1983) as children change from working at a concrete to an abstract level. The poor mathematical ability subjects in this study had not made the transition from concrete to abstract reasoning and actually found it difficult to conciously think internally. This transition is important in mathematics because numbers are abstract ideas and until this is appreciated only procedural skills are likely to develop.

Building up a picture of subjects' number schema was a useful technique in highlighting differences between subjects. The gap in understanding what numbers are and the absence of links between what is known ties in very well with the dependence links between what is known ties in very well with the dependence on procedural knowledge demonstrated by the poor mathematical ability subjects. Describing a schema offers more flexibilty than outlining stages of development such as those given by Piaget (1953) or drawing complex maps and models of skills like those in Siegler and Robinson (1982). Schema description allows for individual differences in skill acquisition and outlines current knowledge. This information may be useful in planning instructional programmes.

10.3 Teaching Mathematics

Generative learning (Wittrock,1974) stresses the importance of prior knowledge in acquiring new expertise so it is important for teachers to have a clear understanding of what children know and do in mathematics. Research studies have shown that children develop informal methods of problem solving based on counting procedures while school taught procedures are mechanically applied to numbers (de Vere, 1989, Ginsburg 1977).

The Beginning School Mathematics Programme (1985) used in junior school classrooms does not consider counting to be an important activity but children are developing their own methods of problem solving based on counting and through counting begin to see the patterns and relationships between numbers that are important in developing an understanding of number.

Children's counting has been studied (Steffe,von Glasersfeld,Richards and Cobb,1983; Siegler and Robinson, 1982;Fuson,Richards and Briars,1982) but more attention could be directed to counting back and in multiples to see how these types of counting tie into number concept development.

This study has shown a difference in information processing between the different ability groups. There clearly exists a need to develop programmes that help children to develop abstract thinking. At the present time children are encouraged to continue with concrete materials to help with computation but little progress is made in changing the way they process information. For many children in our schools only procedural knowledge is learned and applied.

The remedial programme trialled in this study did appear to be changing information processing as subjects stopped counting on fingers and began to use prior knowledge. The results of the tests given did not clearly show the effects of this observed change in behaviour although the groups in the trial programme did show more subjects reaching mastery level than in the control programme. The programme only ran for six weeks and given a longer time more progress should be shown. The teachers involved in the study intend to follow up the ideas included in this programme in future mathematics lessons.

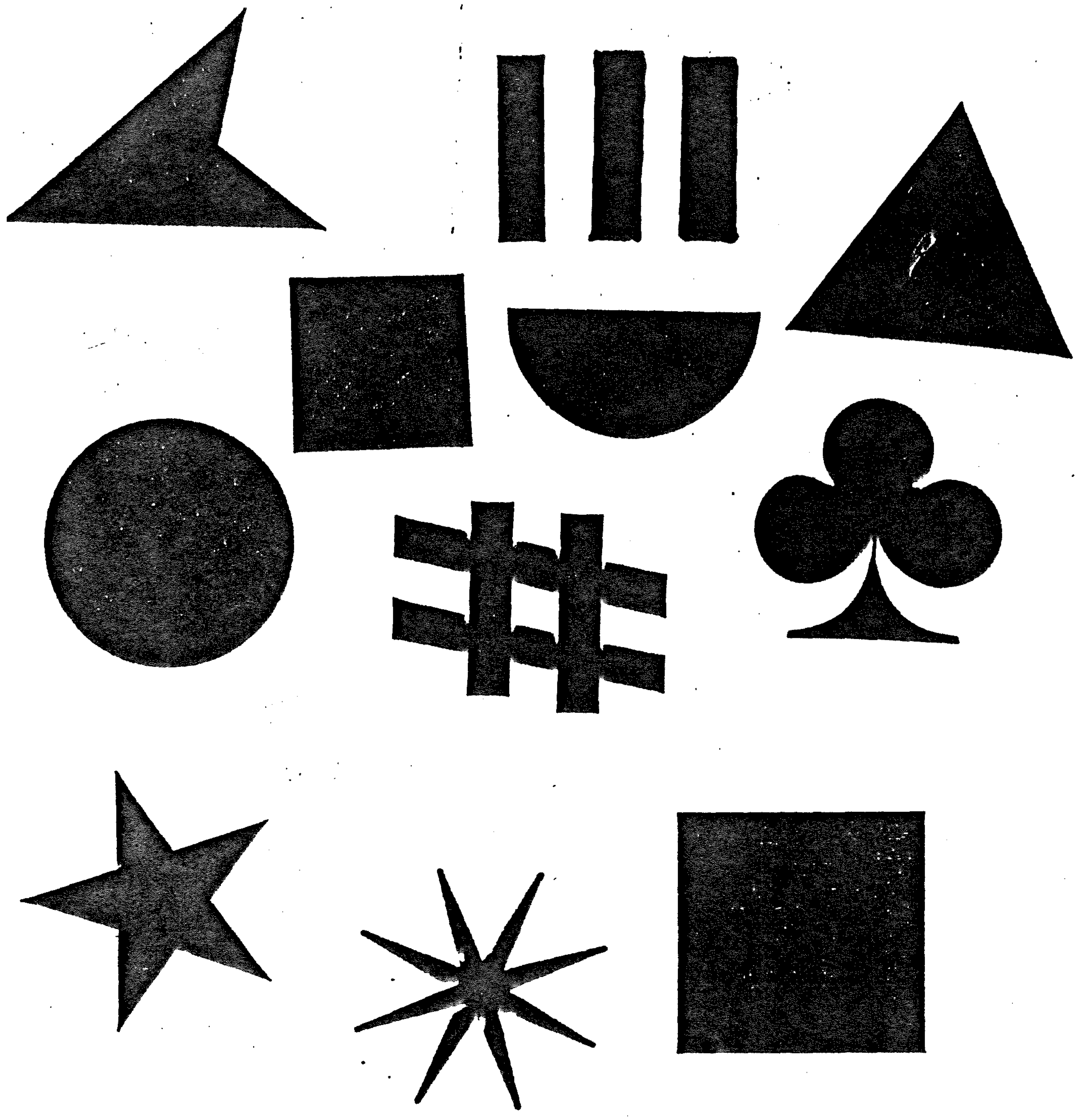
There are several implications for teachers arising from this study including the need to move away from instructional activities that involve procedural knowledge towards activities that develop abstract thinking leading to concept development and improved problem solving skills. This means teachers need to become aware of children's current understanding and informal processing skills in planning programmes that will include activities that require the application of understanding in problem solving situations.

Hughes (1986) saw the problem as an inability to transfer between the concrete real world and the written formal code of school arithmetic. He expressed the need for a meaningful introduction to mathematical symbols. This research supports that point of view but explains the problem in terms of changing from concrete to abstract thinking and away from procedural knowledge towards concept development and application. A programme designed to implement these findings did have some success but more research into programme development is required so that the findings of research can be implemented into the classroom setting.

APPENDIX A

VISUAL TEST PRESENTATION SHAPES AND CARDS

Shapes used in M-Space Test H



Examples of cards used in the Visual Presentation

APPENDIX B

TEST ADMINISTRATION OUTLINE

Normal test conditions apply, as outlined in P.A.T. Manuals.

Materials: Test paper for each student, Instructions and stop watch.

Procedure: Distribute test papers, students write in name age and class level in appropriate spaces and fold paper in half lengthways.

Say, "YOU ARE GOING TO DO A SERIES OF SHORT EXERCISES THAT WILL TEST HOW WELL YOU CAN REMEMBER NUMBERS AND WORK THINGS OUT IN YOUR HEAD. YOU MUST LISTEN CAREFULLY AS EACH QUESTION WILL BE READ ONCE ONLY AND THERE IS A TIME LIMIT FOR WRITING YOUR ANSWERS ON THE SHEET PROVIDED. BEFORE EACH QUESTION IS READ YOU WILL BE TOLD TO FIND THAT QUESTION NUMBER ON YOUR ANSWER SHEET SO THAT YOU ARE READY TO RECORD YOUR ANSWER AS QUICKLY AND ACCURATELY AS YOU CAN ON THE LINE NEXT TO THAT QUESTION NUMBER (Show lines on sheet).

THE SIGNAL TO PICK UP YOUR PENCIL AND WRITE WILL BE WHEN YOU HEAR THE WORD 'GO' AND THE SIGNAL TO PUT YOUR PENCILS DOWN AGAIN IS WHEN YOU HEAR THE WORD 'STOP'. (Practise use of 'go' and 'stop'). THIS IS HOW YOU WILL DO ALL THE QUESTIONS ON THE TEST. LET'S TRY IT NOW. FIND TEST A. NOW FIND QUESTION S.1. PUT YOUR FINGER BY THAT QUESTION . YOU WILL HEAR 'LISTEN' AND THEN THE QUESTION WILL BE READ, THEN 'GO' WHEN YOU WILL PICK UP YOUR PENCILS AND WRITE DOWN YOUR ANSWER AND THEN 'STOP' WHEN YOU WILL PUT YOUR PENCIL DOWN AGAIN READY TO LISTEN TO THE NEXT QUESTION."

Answer any questions and say "WE'LL BEGIN THE TEST NOW."

TEST A COUNTING SEQUENCE

"THIS IS A TEST OF HOW WELL YOU KNOW THE COUNTING NUMBERS. YOU WILL HEAR A NUMBER AND WHEN I SAY 'GO' YOU ARE TO WRITE DOWN THE NEXT COUNTING NUMBER, THAT IS THE NUMBER JUST AFTER THE NUMBER YOU HEAR. HERE IS AN EXAMPLE TO SHOW YOU HOW TO DO THIS TEST. YOU DO NOT NEED TO WRITE ANYTHING THIS TIME.

IF I SAY 2, THE NUMBER JUST AFTER 2 IS (Ask for response) SO YOU WOULD WRITE 3 ON YOUR ANSWER SHEET BECAUSE 2 IS THE NEXT COUNTING NUMBER AFTER 2.

NOW YOU TRY THIS EXAMPLE:

FIND TEST A, FIND QUESTION S.1.

LISTEN, THREE, GO (10 Sec. to record) STOP.

Q1. 14	Q2. 19
Q3. 26	Q4. 74
Q5. 39	Q6. 89
Q7. 200	Q8. 500
Q9. 209	Q10. 509

(Check answers and correct filling of sheets.)

<u>TEST B</u> BACKWARD COUNTING SEQUENCE

IN THIS TEST YOU ARE TO WRITE DOWN THE COUNTING NUMBER THAT IS <u>JUST BEFORE</u> THE NUMBER YOU HEAR. IF I SAY 8,YOU WOULD WRITE (ask for response) ON YOUR SHEETS.

NOW TRY THIS EXAMPLE YOURSELF:

FIND TEST B
FIND QUESTION S.2.
LISTEN, SIX, GO (10 Sec.) STOP.

Q11. 15	Q12. 19
Q13. 48	Q14. 74
Q15. 40	Q16. 89
Q17. 300	Q18. 500
Q19. 110	Q20. 510

<u>TEST C</u> ADDITION OF TEN

IN THIS TEST YOU ARE TO <u>ADD TEN</u> TO THE NUMBER YOU HEAR.
IF I SAY 'TWO' YOU WOULD WRITE (Ask for response) 12 ON YOUR SHEET.

NOW TRY THIS EXAMPLE YOURSELF:

FIND TEST C.
FIND QUESTION S.3.
LISTEN, FIVE ,GO (5 SEC.) STOP.

Q21. 4	Q22. 9
Q23. 16	Q24. 52
Q25. 350	Q26. 190
Q27. 503	Q28. 109
Q29. 294	Q30. 691

TEST D SUBTRACTION OF TEN

IN THIS TEST YOU ARE TO SUBTRACT TEN FROM THE NUMBER YOU HEAR.
IF I SAY, 19, YOU WOULD WRITE (Ask for response) 9 ON YOUR SHEET.

NOW TRY THIS EXAMPLE FOR YOURSELF.

FIND TEST D.
FIND QUESTION S.4.
LISTEN, 13, GO (5 SEC) STOP.

Q31. 12	Q32. 17
Q33. 36	Q34. 87
Q35. 275	Q36. 461
Q37. 500	Q38. 700
Q39. 407	Q40. 605

TEST E PATTERNS

IN THIS TEST YOU ARE TO LISTEN CAREFULLY TO THE FOUR NUMBERS I SAY AND THEN YOU ARE TO WRITE THE NEXT TWO NUMBERS THAT YOU THINK WILL FOLLOW MY NUMBERS.
IF I SAY, 1, 2, 3, 4 YOU WOULD WRITE (Ask for response) 5, 6 on your sheet.

NOW TRY THIS EXAMPLE FOR YOURSELF:

FIND TEST E
FIND QUESTION S.5.
LISTEN, 10 -20-30-40-GO (10 SEC.) STOP.

Q41. 4-7-4-7	Q42. 9-1-9-1
Q43. 17-27-37-47	Q59-57-55-53
Q45. 11-22-33-44	Q46. 50-45-40-35
Q47. 98-87-76-65	Q48. 91-82-73-64
Q49. 2-5-8-11	Q50. 98-94-90-86

TEST F DIGIT SPAN

IN THIS TEST YOU ARE TO REMEMBER THE NUMBERS I SAY AND WRITE THEM DOWN IN THE SAME ORDER AS YOU HEAR THEM.
IF I SAY, 3,4 YOU WOULD WRITE (Ask for response) 3,4..ON YOUR SHEET.

NOW TRY THIS EXAMPLE FOR YOURSELF:

FIND TEST F.
FIND QUESTION S.6.
LISTEN, 4-1-8 GO (10 SEC.) STOP.

Q51. 3-8-6	Q52. 6-1-2
Q53. 3-4-1-7	Q54. 6-1-5-8
Q55. 8-4-2-3-9	Q56. 5-2-1-8-6
Q57. 3-8-9-1-7-4	Q58. 7-9-6-4-8-3
Q59. 5-1-7-4-2-3-8	Q60. 9-8-5-2-1-6-3

TEST G DIGIT SPAN BACKWARDS

IN THIS TEST YOU ARE TO REMEMBER THE NUMBERS I SAY BUT YOU ARE TO WRITE THEM IN THE OPPOSITE ORDER TO THE WAY YOU HEAR THEM, THAT IS WRITE THE LAST NUMBER YOU HEARD FIRST AND END WITH THE FIRST NUMBER YOU HEARD.
IF I SAY 9, 6 YOU WOULD WRITE (Ask for response) 6, 9 ON YOUR SHEET.

NOW TRY THIS EXAMPLE FOR YOURSELF:

FIND TEST G.
FIND QUESTION S.7.
LISTEN, 4-6 GO (10 SEC.) STOP.

Q61. 2-5	Q62. 6-3
Q63. 5-7-4	Q64. 2-5-9
Q65. 7-2-9-6	Q66. 8-4-9-3
Q67. 4-1-3-5-7	Q68. 9-7-8-5-2
Q69. 1-6-5-2-9-8	Q70. 3-6-7-1-9-4

TEST H M-SPACE

IN THIS TEST YOU ARE TO COUNT THE NUMBER OF TIMES YOU HEAR EACH ONE OF A SERIES OF DIFFERENT SOUNDS, REMEMBER THE TOTALS YOU COUNTED FOE EACH DIFFERENT SOUND AND WRITE DOWN THE TOTALS IN THE SAME ORDER AS YOU HEARD THE SOUNDS.ALL THE SOUNDS AND FURTHER INSTRUCTIONS ARE ON A TAPE. LISTEN TO THE TAPE AND DO EXACTLY AS YOU ARE TOLD. (Switch on tape).

NOW TRY THIS EXAMPLE FOR YOURSELF:

FIND TEST H

FIND QUESTION S.8.

LISTEN, 3-4 GO STOP.

Q71. 5-2	Q72. 8-3
Q73. 4-7-2	Q74. 3-6-4
Q75. 6-3-5-2	Q76. 7-4-2-6
Q77. 3-5-4-7-2	Q78. 2-7-3-5-4
Q79. 7-4-3-5-2-6	Q80. 4-2-5-3-6-7

TEST J ADDITION EXAMPLES

IN THIS TEST I WILL READ A SERIES OF NUMBER SENTENCES TO YOU AND EACH SENTENCE HAS A MISSING NUMBER IN IT. IN PLACE OF THAT MISSING NUMBER I WILL SAY THE WORD 'SOMETHING'.YOU ARE TO WRITE THE MISSING NUMBER ON YOUR SHEET.

IF I SAY TWO PLUS ONE EQUALS SOMETHING YOU WOULD WRITE (Ask for response) 3 ON YOUR SHEET.

NOW TRY THIS EXAMPLE FOR YOURSELF:

FIND TEST J.

FIND QUESTION S.9.

LISTEN, SIX PLUS TWO EQUALS SOMETHING, GO (IO SEC.) STOP.

Q81. 3+4=[ ]	Q82. 5+2=[ ]
Q83. 4+9=[ ]	Q84. 5+6=[ ]
Q85. 9+[ ]=12	Q86. 5+[ ]=13
Q87. [ ]+8=15	Q88. [ ]+9=14
Q89. 2+9=4+[ ]	Q90. 6+8=5+[ ]

TEST K SUBTRACTION EXAMPLES

IN THIS TEST EACH NUMBER SENTENCE AGAIN HAS A MISSING NUMBER AND I WILL SAY THE WORD 'SOMETHING' IN PLACE OF THAT MISSING NUMBER. YOU ARE TO WRITE THE MISSING NUMBER ON YOUR SHEET.
IF I SAY FIVE MINUS THREE EQUALS SOMETHING YOU WOULD WRITE (Ask for response) TWO ON YOUR SHEET.

NOW TRY THIS EXAMPLE:

FIND TEST K
FIND QUESTION S.10.
LISTEN, FIVE MINUS THREE EQUALS SOMETHING, GO (10 SEC.) STOP.

Q91. 9-5=[]	Q92. 10-3=[]
Q93. 17-8=[]	Q94. 11-7=[]
Q95. 15-[]=9	Q96. 12-[]=4
Q97. []-6=7	Q98. []-3=8
Q99. []-7=1+7	Q100. []-6=5+4

TURN OVER YOUR ANSWER SHEET.
YOU HAVE JUST HEARD TWENTY NUMBER SENTENCES READ OUT. NOW I WANT YOU TO WRITE DOWN AS MANY DIFFERENT NUMBER SENTENCES AS YOU CAN THINK OF. YOU MUST WRITE WHOLE EQUATIONS THAT HAVE NUMBERS ON EACH SIDE OF THE EQUALS SIGN. THIS IS A CHANCE FOR YOU TO SHOW ME HOW MUCH YOU KNOW ABOUT NUMBERS. (Answer questions).
YOU HAVE FIVE MINUTES TO WRITE AS MUCH AS YOU CAN. GO.
(After 5 min.) STOP.
COLLECT PAPERS.

APPENDIX C

COMPLETED ANSWER SHEET

Test A

		S.1	4
Q.1	15	Q.2	20
Q.3	27	Q.4	75
Q.5	40	Q.6	90
Q.7	201	Q.8	501
Q.9	210	Q.10	~~50~~ 510

5

Test F

		S.6	4,1,8
Q.51	3,8,6.	Q.52	6,1,2.
Q.53	3,4,1,7.	Q.54	6,1,5,8
Q.55	8,4,2,3,9	Q.56	5,2,1,8,6.
Q.57	3,8,9,1,7,4	Q.58	7,9,6,4,8,3
Q.59	5,1,7,4,2,3,8.	Q.60	9,8,5,2,1,6,3

4

Test B

		S.2	5
Q.11	14	Q.12	17
Q.13	47	Q.14	62
Q.15	39.	Q.16	69
Q.17	299	Q.18	799.
Q.19	109.	Q.20	309.

5

Test G

		S.7	6,4.
Q.61	52,	Q.62	3,6.
Q.63	4,75.	Q.64	9,5,2
Q.65	6,2,79.	Q.66	4,8,9,3.
Q.67	3	Q.68	25738.
Q.69	—	Q.70	—

2.5

Test C

		S.3	15
Q.21	14	Q.22	19
Q.23	26	Q.24	62
Q.25	360	Q.26	200
Q.27	513.	Q.28	119
Q.29	304	Q.30	701

5

Test H

		S.8	3,4.
Q.71	5,2.	Q.72	8,2.
Q.73	4,6	Q.74	3,6,5.
Q.75	5,3,5,2	Q.76	—
Q.77	—	Q.78	—
Q.79	—	Q.80	4,53,7.

0.5

Test D

		S.4	3
Q.31	2	Q.32	7
Q.33	26	Q.34	77
Q.35	265.	Q.36	451
Q.37	570	Q.38	690
Q.39	397	Q.40	595

5

Test J

		S.9	3
Q.81	7	Q.82	7
Q.83	13	Q.84	11
Q.85	3	Q.86	8
Q.87	7	Q.88	~~5~~ 5
Q.89	7	Q.90	—

4.5

Test E

		S.5	50,60
Q.41	4,7	Q.42	9,1
Q.43	57,67	Q.44	51,49
Q.45	55,66.	Q.46	30,25
Q.47	54,43	Q.48	—
Q.49	14,17	Q.50	82,78.

4.5

Test K

		S.10	2
Q.91	4	Q.92	7
Q.93	9	Q.94	4
Q.95	6	Q.96	8
Q.97	13.	Q.98	11
Q.99	15	Q.100	15

5

APPENDIX D

EXAMPLE OF PROGRAMME INSTRUCTIONS

Mathematics Programme T

Aim:To develop skills of working at an abstract level in mathematics.

Specific Objectives for Subjects:

To think in your head.

To use prior knowledge in solving new problems.

To apply these skills in problem solving situations.

Introduction

Over the next six weeks we are going to take part in a daily programme that will help improve your maths. The way you are going to do this is to learn how to ask yourself questions and then answer them. Obviously if everyone is asking questions all the time it will soon get very noisy and everyone will find it difficult to concentrate on their work. To overcome this problem you have to be able to talk to yourself silently, that is talk inside your head rather than out loud.

This is not only important, not only so that the classroom is quiet, but because you can use all the knowledge you have stored away in your brain. everyone has some knowledge about numbers in their memory but you can only use it if you ask for it, like withdrawing money from a bank. So you have to learn how to ask yourself questions.

Now let's find out about talking in our heads. Put your finger on your throat and find your voice box or larynx. When I say, "Go," count to ten aloud and you will feel it vibrate. At the same time your lips and tongue are moving. (Try this activity). Now I want you to count to ten without moving your lips, tongue or voice box. Try counting again, silently this time. (Find out which children can or can't do it.) Try it with a partner and watch for their lips moving. (Suggest to children that this activity can be practised throughout the day and at home.)

For those having difficulty suggest closing their mouth, shutting eyes and putting hands over ears to avoid other distractions.

Finally everyone ask themselves this question ,"What do I know about addition?" Think about this question and we'll share our answers tomorrow. Remind children to practise thinking in their heads by counting to themselves. How far can they count in 30 seconds, one minute...?

Mathematics Programme M

Aim: To develop recall of basic addition and subtraction facts and problem solving skills.

Specific Objectives for Subjects:

To list ways of learning facts.

To practise addition/subtraction facts in examples.

To use addition/subtraction facts in games and puzzles.

To use three or five step method of problem solving.

Introduction

Over the next six weeks we are going to take part in a daily programme that we help you to learn your addition and subtraction facts like you learn your times tables. This will help you when solving addition and subtraction problems as you will be able to recall the facts quickly without having to count or work it out.

Today we are going to discuss ways of learning information. (Ask for suggestions, especially from those children who know their facts well.) Suggest reciting facts, writing facts, testing yourself, finding out which facts you know well and which ones you need towork on.

Working in pairs ask each other five facts. See how quickly you can answer correctly. Count if you have to. Practise facts throughout the day and at home.

APPENDIX E

GAME AND PUZZLE INSTRUCTIONS

Activities used in Programme M and T

Basic Facts Noughts and Crosses

Divide children playing into two teams.

Draw a noughts and crosses grid on the blackboard.

Write facts in the grid spaces.

5+8	7+0	6+9
0+3	4+9	2+1
7+8	9+2	6+8

Play the game as noughts and crosses:

One team is noughts, the other crosses.

Each team has alternate turns at playing.

To put a nought or cross on the board a team member has to correctly answer a chosen fact on the playing grid.

Team members are allowed only one turn at playing until all the team has had their turn.

If the answer given is incorrect then no score is made for that turn and the other team has their turn.

The winning team is the first to complete a row of noughts or crosses.

The game may be played with subtraction, multiplication or division facts.

Basic Facts Housie

Children write down ten different numbers of their choice.

Numbers should be from 0 to 18.

The teacher or an able child is the first caller.

The caller prepares a list of facts to be tested.

Addition and subtraction facts are called out one at a time.

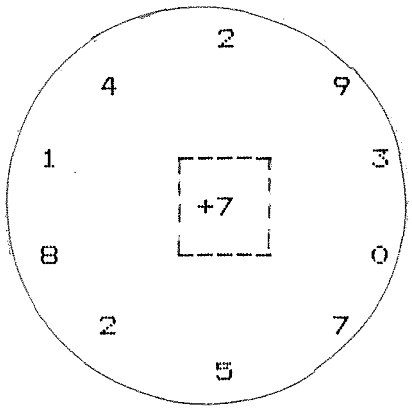
If the solution for that fact appears on a list of chosen numbers it is crossed off.

The game continues until one player has crossed off all ten numbers.

The numbers are checked against the facts given.

The winner may have the next turn as caller.
The difficulty of the game may be increased by extending the limit for chosen numbers or by the caller using a variety of question types. This could include any topic from the syllabus, for example, "The face value of the 5 in the numeral 458?"

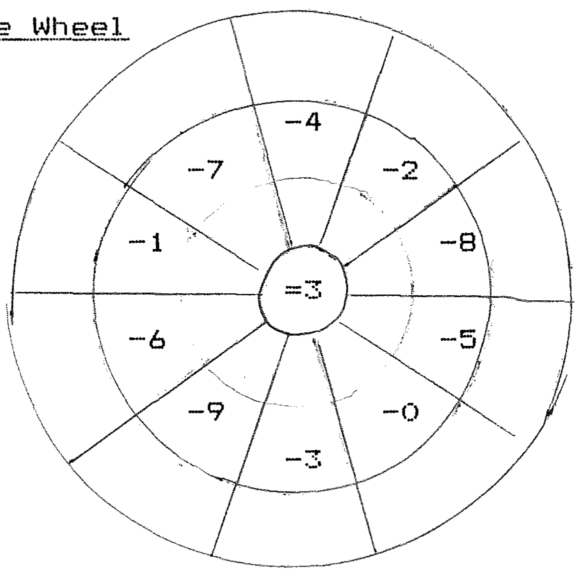
Beat the Clock



Draw a 'clock face' with the ten digits arranged in random order around it.
Draw a square in the centre of the clock and write a number and addition sign in it.
The game is played by adding the central number to each of the outer digits in turn.
A time limit is set for each child to beat the clock.
Change the central number at the end of each attempt.
A successful turn may allow another turn at a different number.

The game may be played with multiplication facts and also subtraction and division facts if the outer digits are chosen appropriately.

Complete the Wheel



This game is like 'Beat the Clock' but this time the outer section of a wheel is empty and children have to find the number to fill in the space in each sector of the wheel.
The wheel is drawn on a blackboard and the game played the same as 'Beat the Clock'.
The game can be varied by making the unknown number appear in the central ring or the middle ring of the wheel.
Aternatively the game may be played using addition, multiplication or division facts.

Alphabet Codes

1. Write the alphabet.
2. Starting with A as 1, B as 2 ...Z as 26 number each letter of the alphabet.
3. Use this code to complete the following activities:

a. Write your name in code and find the sum of the 'letters.'
b. Write down five friends name and find the sum for their names. Does the longest name have the greatest sum?
c. Write names of countries, days, months, animals... and find their sum.
d. Write a word or message in code for a friend to de-code.

Counting Letters

1. Select a passage of 100 words.
   Count the number of times each vowel occurs.
   Discuss findings with other children.
   Express your results as fractions or percentages (older children).
2. Using groups of words like names of days, months... answer questions about the letters in the names.
   Examples: How many months end with the letter 'y'?
   How many days have 6 letters in their name?

What am I?

Children make clues to describe a number.
Other children try to name the number described.
The aim is to write clues that describe only one number.
Example: I am odd.
I am less than 10 and greater than 5.
If you turn me upside down I am a different number.

If you look at me in a mirror I am a letter.

Answer:9

Test Activities

1. Oral test ,facts and word problems.

8. Speed test, examples written on the board,record answers.

3. Grids: ten by ten, random digit order, number array .

+	3	6	1	8	0	5	2	7	9
4									
0									
7									
3									
9									
5									
1									
8									
2									

APPENDIX F

TEACHER EVALUATION SHEET

I would like you to be completely honest in evaluating the programme. I am looking for positive and negative comments so that I can make appropriate changes and suggestions for a longer programme. Fill in as much as you can but if you wish you may leave some responses if you have no comment on that aspect of the programme. I have only given broad headings so that I do not influence your response. (You may continue on the back of the page) Thank you for your co-operation

Organisation

Grouping - a help to have this done for us.
Yes - Groups are necessary with older chn.

Time - Some lessons too long but as a warm-up to Maths lesson these can be continued if required following day.

Answer Books Good idea to use graph paper.
How about normal Maths bks.

Content

Instructions - teachers instructions not always clear - chns good.
Purpose to lesson should be given to test appropriate lessons.

Activities - Good for encouraging thinking.

Continuity/Aims - ✓

Outcome of participating in the programme

For children - more logical thinking & extra mental work which is lacking in Maths texts.

For teachers - emphasises importance of group wk at Standard level.

For future planning - ↑

Using the Programme - were any activities useful for :-

Diagnostic purposes - Yes - weekly quick grids note improvement.

Maintenance - continuing mental work.

Introducing New Concepts / clearing misconceptions
NO - wouldn't do it so quickly. Yes.

Any other Comments Main benefit is for encouraging individual [illegible] problems of chn in "think[illegible]"

REFERENCES

Alton Lee,A.(1984) Understanding,Learning and Teaching:Ph.D Thesis, University of Canterbury.

Atkinson,R.C. and Shiffrin,R.M.(1968) Human Memory:A Proposed System and its Control Processes.In: K.W. Spence and J.T. Spence (eds.) The Psychology of Learning and Motivation Vol 2.Academic.

Baddley,A.D. and Hitch,G. (1974) Working Memory. In:G.H. Bower (ed) The Pyschology of Learning and Motivation,18, Academic Press.

Baroody, A.J. (1983) The Case Of Adam: A Specific Evaluation of a Math Learning Disability. Paper presented at annual meeting of the American Educational Research Association, Montreal.

Baroody, A.J. (1987) The Development of Counting Strategies for Single Digit Addition. Journal for Research in Mathematics Education, 18,2, 141-157.

Brainerd.C.J. The Origins of the Number Concept.N.Y.Praeger.

Brown and DeLoache. (1978) In R.S.Siegler (ed.), Children's Thinking:What Develops? Hillsdale,NJ:Erlbaum

Carpenter, T.P., Moser, J.M. (1982) The Development of Addition and Subtraction Problem-Solving Skills. In T.P.Carpenter, J.M.Moser & T.A.Romberg (eds.) Addition and Subtraction: A Cognitive Perspective Hillsdale, N.J. Erlbaum.

Case,R. (1978)Intellectual Development from Birth to Adulthood: A Neo-Piagtian Interpretation.In R.S.Siegler (ed.), Children's Thinking:What Develops? Hillsdale,NJ:Erlbaum

Case,R. (1982).General Developmental Influences on the Acquisition of Elementary Concepts and Algorithms in Arithmetic.In T.P.Carpenter,J.M.Moser &T.A.Romberg(eds.), Addition and Subtraction:A Cognitive Perspective.Hillsdale, NJ:Erlbaum

Case,R., Kurland,M. and Goldberg,J. (1982) Operational Efficiency and the Growth of Short-Term Memory. Journal of Experimental Child Psychology 33, 386-404.

Ceci, S.J. (1984) A Developmental Study of Learning Disabilities and Memory. Journal of Experimental Child Psychology 38, 352-371.

Chase,W.G., & Simon,H.A. (1973a) Perception in Chess. Cognitive Pyschology,4

Chase, W,G. and Ericsson,K.A. (1981) Skilled Memory . In: J.R.Anderson (Ed.) Cognitive Skills and Their Acquisition,Lawrence Erlbaum,1981.

Chi,M.T.H. (1978) Knowledge Structures and Memory Development. In:R.S.Siegler (Ed.) Children's Thinking. What Develops? Hillsdale,N.J.

Clay,M.M. (1979).The Early Detection of Reading Difficulties: A Diagnostic Approach.Auckland.Heinemann.

Clements, D.H. (1984). Training Effects on the Development and Generalisation of Piagetian Logical Operations and Knowledge of Number. Journal of Educational Psychology 76, 766-776.

Cohen,G.,Eysenck,M.W. and LeVoi,M.e. (1986) Memory A Cognitive Approach. Open University Press.

Denvir,B. and Brown,M. (1986) Understanding of Number Concepts in Low Attaining 7-9 Year Olds; Part 1 Development Descriptive Framework and Diagnostic Instrument. Educational Studies in Mathematics, 17,1,13-35.

Denvir,B. and Brown,M.(1986) Understanding of Number Concepts in Low Attaining 7-9 Year Olds; Part II.The Teaching Studies Educational Studies in Mathematics, 17,2,143-164.

Department of Education (1985). Beginning School Mathematics. Wellington: Government Printer.

de Vere,P.M. (1989) Do Children Use Methods taught in Schools When Solving Arithmetic Problems? M.A. Research Project. University of Canterbury.

Duffy, D. and Kernahan P.(1987) Daily Perceptual Motor Training Activities.Christchurch Teachers' College.

Flavell,J.H. (1978) Comments on Papers.In:R.S.Siegler (Ed.) Children's thinking. What Develops? Hillsdale,N.J.

Fuson, K.C., Richards, J. & Briars, D.J. (1982) The Acquisition and Elaboration of the Number Word Sequence. In C.J.Brainerd (ed.), Children's Logical and Mathematical Cognition: Progress in Cognitive Development Research. New York: Springer-Verlag.

Gelman, R. & Gallistel, C.R. (1978). The Child's Understanding of Number. Cambridge, MA: Harvard Press.

Ginsburg, H.P. (1977). Children's Arithmetic: The Learning Process. New York: D. Van Nostrand.

Ginsburg, H.P. (1982). Children's Arithmetic: How They Learn It and How You Teach It. Austin, Texas: Pro-ed.

Ginsburg, H.P. (1982) Contexts of Culture, Social Class, and Race. In T.P. Carpenter, J.M. Moser, & T.A. Romberg (eds.), Addition and Subtraction: A Cognitive Perspective. Hillsdale, N.J. Erlbaum

Groen , G. & Parkman,J. (1972). A Chronrmetric Analysis of Simple Addition Pyschological Review 79,329-343.

Groen, G. & Resnick, L.B. (1977). Can Preschool Children Invent Addition Algorithms? Journal of Educational Psychology, 69, 645-652.

Hebb, D.O. (1961) Distinctive features of Learning in the Higher Animal.In: J.F.Delafresnaye (ed) Brain Mechanisms and Learning.Blackwell.

Hope,J.A. (1987) A Case Study of a Highly Skilled Mental Calculator. Journal for Research in Mathematics Education 18, 331-343.

Houlihan,D.M. and GinsburgH.P. (1981) The Addition methods of First and Second Grade Children.Journal for Research in Mathematics.12. 95-106.

Hughes, M. (1983) What is Difficult about Learning Arithmetic? In M. Donaldson, R. Grieve & C. Pratt (eds), Early Childhood Development And Education. Oxford: Basil Blackwell.

Hughes, M. (1986). Children and Number: Difficulties in Learning Mathematics. Oxford: Basil Blackwell.

Hulme,C., and Tordoff,V. (1989) Working Memory Development: The Effects of Speech Rate, Word Length, and Acoustic Similarity on Serial Recall.Journal of Educational Psychology 42,72-87.

Hunter,I,M.L. (1977) An Exceptional Memory. British Journal of Psychology, 68,155-164.

Markus, H. (1980) The Self in Thought and Memory. In: D.M. Wegner and R.R. Vallacher (eds.)The Self in Social Psychology.N.Y.

Miller,G.A. (1956) The Magical Number Seven Plus or Minus Two. Pyschological Review 63, 81-97.

Newman,R.S.,Friedman,C.A. and Gockley,D.R. (1987) Children's Use of Multiple Counting Skills: Adaptation to task factors. Journal of Educational Psychology 44, 268-282.

Piaget,J. (1953) How Children Form Mathematical Concepts. Scientific American, 189, 74-79.

Pollio,H.R. and Whitacre,J.D. (1970) Perceptual and Motor Skills, 30, 167-174.

Popham,W.J (1967) Educational Statistics Harper and Row..

Rastak,R.M.(1984) The Brain. Bantam.

Reid,N.A.& Hughes,D.C. (1974) Progressive Achievement Tests: Mathematics. N.Z.C.E.R. Whitcoulls.

Resnick, L.B. (1982) Syntax and Semantics in Subtraction, In T.P.Carpenter, J.M. Moser & T.A. Romberg (eds) Addition and Subtraction: A Cognitive Perspective. Hillsdale, N.J.Erlbaum.

Saxe,G.B.(1985) Effects of Schooling on Arithmetical Understandings.Journal of Educational Psychology,77,503-513.

Schaeffer,b., Eggleston,V.H.& Scott,J.L. (1974) Number Development in Young Children.Cognitive Pyschology,6, 357-379.

Siegler, R.S. & Robinson, M. (1982) The Development of Numerical Understandings. In H.W. Reese & L.P. Lipsett (eds) Advances in Child Development and Behaviour. Vol. 16 New York: Academic Press.

Steffe, L.P. Von Glasersfeld, E., Richards, J. & Cobb, P. (1983). Children's Counting Types: Philosophy, Theory and Application. Praeger.

Steffe,L.P. (1983) Children's Algorithms As Schemes. Educational Studies in Mathematics. 14,2,110-125.

Steinberg,R.M. (1985) Instruction on Derived facts Strategies in Addition and Subtraction. Journal for Research in Mathematics Education .16, 5, 337-355.

Stillings,N.A.,and Feinstein,M.H.Et al., (1987) Cognitive Science M.I.T. Press.

Wechsler,D. (1974) Manual for the Wechsler Intelligence Scale for Children-Revised. N.Y.

Wittrock,M.C. (1974).A Generative Learning Model of Mathematics Learning.Journal for Research in Mathematics Education.5.

Young-Loveridge,J. (1987) The development of Children's Number Concepts.University of Canterbury